# LEARNING MATERIAL OF

# HYDRAULICS & IRRIGATION ENGINEERING

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**&**

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## Hydraulics

Hudour (Greek word)
↓
Water

Hydraulics may be defined as the branch of engineering science, which deals with the water at rest or at motion.

### Fundamental units :-

The measurement of physical quantities is one of most important operations in engineering

Fundamental units are

> Length --- M
> Mass --- Kg
> Time --- sec

### Derived unit :-

Some units which are derived from the fundamental units are called derived units.

Ex:- Area, velocity, Acceleration, displacement, pressure etc.

$$\boxed{\text{Pressure} :- \frac{N}{m^2} = \frac{\text{Force}}{\text{Area}}}$$

### S.I unit

Density = $\frac{Kg}{m^3}$

Force - Newton(N) ⇒ $\frac{Kg \cdot m}{sec^2}$

Pressure - $N/m^2$

Area - $m^2$

Velocity - $m/s$

Acceleration = $\frac{\text{velocity}}{\text{Time}}$ - m/sec²

M·K·S - Mass, Kilogram, second

C·G·S - Centimetre, Gram, Second

$$\frac{kg \cdot m}{sec^2} \rightleftarrows \frac{1000\ gm \times 100\ cm}{sec^2}$$

$$= 10^5 \frac{gm \cdot cm}{sec^2}$$

$$\boxed{1N = 10^5\ \text{dyne}}$$

C·G·S unit

1 kg = 1000 gm

= $10^3$ K

$10^2 \rightarrow$ hecto

$10^6 \rightarrow$ Mega

$10^9 \rightarrow$ Giga

$10^{12} \rightarrow$ Tera

$10^{-1} \rightarrow$ Desi

$10^{-2} \rightarrow$ centi

$10^{-3} \rightarrow$ Milli

$10^{-6} \rightarrow$ micro

$10^{-9} \rightarrow$ Nano

$10^{-12} \rightarrow$ Pico

Liquids & their properties :-

(a) The properties of liquid are

(i) Density

(ii) specific weight

(iii) specific gravity

(iv) surface Tension

(v) Cappilarity

(vi) viscosity

(vii) Compressibility

## Density :- ($\rho$)

→ The density of a liquid may be defined as the mass per unit volume and per at a standard temperature of a pressure.

→ It is also called as mass density

Density ($\rho$) = $\frac{Mass}{Volume} = \frac{M}{V}$

$\boxed{\rho = \frac{M}{V}}$ $\frac{kg}{m^3}$

→ unit of Density = $kg/m^3$ / $gm/cm^3$

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1Q. If $0.5\,m^3$ of a certain oil has a mass of 2 tonnes find it mass density.

Sol? Given data

volume = $0.5 m^3$

Mass = 2 tonnes = $2\times10^3$ kg

$\boxed{1 \text{ tonnes} = 1000 kg}$

Mass density or density = $\frac{Mass}{Volume}$

$= \frac{2\times10^3 kg}{0.5 m^3}$

$= 800\, kg/m^3$

$1 kN = 1 \times 10^3 N$

$$\boxed{1 kN = 10^3 N}$$

$$10^3 \frac{kg\ m}{sec^2}$$

$$\boxed{1 N = \frac{kg\ m}{sec^2}}$$

**3Q** If the volume of specific liquid is $2.4 m^3$ and that of weight is 1200 N then calculate the specific weight of that liquid.

**Sol^n** Given data

volume $= 2.4 m^3$

weight $= 1200 N$

$$\text{specific weight} = \frac{\text{weight}}{\text{volume}} = \frac{1200 N}{2.4 m^3}$$

$$= 500 N/m^3$$

**4Q** Calculate density of liquid if volume of that liquid is $2.4 m^3$ and weight $= 1200 N$

**Sol^n** Volume $= 2.4 m^3$

weight $= 1200 N$

weight $=$ Mass $\times$ Acceleration due to gravity

$$M = \frac{\text{weight}}{\text{Acceleration due to gravity}}$$

$M = \frac{W}{\ }$

$$M = \frac{1200N}{9.81m/sec^2} = 122.32\ kg$$

$$\rho = \frac{mass}{volume}$$

$$= \frac{122.32kg}{2.4m^3} = 50.96\ kg/m^3$$

5Q In an experiment, the weight of 2.5m³ of a certain liquid was found to be 18.75 kN. find the specific weight of the liquid and also find it's density.

Given data

volume $= 2.5m^3$

weight $= 18.75KN$

$$\text{specific weight } (w) = \frac{weight}{volume} = \frac{18.75}{2.5} = 7.5kN/m^3$$

$$weight = Mg$$

$$\Rightarrow m = \frac{w}{g} = \frac{18.75kN}{9.81m/sec^2} = 1.91131kg \times 10^3 kg$$

$$= 1911.31\ kg$$

$$density\ (\rho) = \frac{M}{V} = \frac{1911.31\ kg}{2.5m^3}$$

$$= 764.524\ kg/m^3$$

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## specific Gravity :-

> It may be defined as the ratio of specific weight of given fluid to that of specific weight of the pure water at standard temperature.

> $$\boxed{\text{Specific gravity} = \frac{w\,\text{fluid}}{w\,\text{water}}}$$

**Q** Find the specific gravity of an oil whose specific weight is $7.85\ KN/m^3$.

**Soln** specific weight of oil ($w_{oil}$) $= 7.85\ KN/m^3$

specific weight of water ($w_{water}$) $= 9.81\ KN/m^3$

$$\text{specific gravity} = \frac{w_{oil}}{w_{water}}$$

$$= \frac{7.85\ KN/m^3}{9.81\ KN/m^3}$$

$$= 0.8$$

Formula

Weight (W) = Mass × Acceleration due to gravity

$\Rightarrow W = m \cdot g$

$\Rightarrow w \times \cancel{V} = \rho \times V \times g$

$\Rightarrow w = \rho \cdot g$

$w = \frac{W}{V}$

$\Rightarrow W = w \times V$

$\rho = \frac{m}{V}$

$\Rightarrow m = \rho \times V$

Form

$w_{water} = \rho_{water} \times g$

$= 1000\ kg/m^3 \times 9.81\ m/sec^2$

$= 9.81 \times 10^3\ kg/m^3\ m/sec^2$

$w_{water} = 9.81\ kN/m^3$

$10^3 = kilo$

Q A vessel of $4m^3$ volume contains an oil, which weighs 30.2 kN Determine the specific gravity of oil.

Sol^n Given data:-

volume $= 4\ m^3$

weight $= 30.2\ kN$

specific weight $= \frac{30.2}{4} = 7.5\ kN/m^3$

specific gravity $= \frac{7.5\ kN/m^3}{9.81\ kN/m^3}$

$= 0.769 \simeq 0.77$

Q Determine the mass density of an oil if 3 tonnes of the oil occupies a volume of $4m^3$.

Given data:-

volume $= 4m^3$

Mass(M) = 3 tonnes

1 t = 1000 kg $= 3 \times 10^3$ kg

Mass density = $\frac{Mass}{volume}$ = $\frac{3000 \text{ kg}}{4 m^3}$ = 750 $kg/m^3$

Q. A certain liquid occupying a volume of $1.6 m^3$, weight 12.8 KN. what is the specific weight of the liquid.

Soln Given data:

volume = $1.6 m^3$

weight = 12.8 KN

specific weight of liquid = $\frac{\text{weight of liquid}}{\text{volume of liquid}}$

$= \frac{12.8 \text{ KN}}{1.6 m^3}$

$= 8 \text{ KN}/m^3$

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$\rho_{oil} = \frac{M_{oil}}{V_{oil}}$  $w_{oil} = \rho_{oil} \times g$

$\rho_{water} = \frac{M_{water}}{V_{water}}$

$\boxed{w = \rho \cdot g}$  = $9.81 m/sec^2$

$\rho_{water} = 1000 kg/m^3 = 1 g/mL$

8 A container of volume $3.0\,m^3$ has has $25.5$ kN of an oil, find specific gravity & Mass density of oil.

Given data:—

volume $= 3.0\,m^3$

weight $= 25.5$ KN

$$\text{specific weight of oil} = \frac{\text{weight of oil}}{\text{volume of oil}}$$

$$= \frac{25.5\ \text{KN}}{3.0\,m^3} = 8.5\ \text{KN}/m^3$$

$$\text{specific gravity of oil} = \frac{w\ \text{oil}}{w\ \text{water}}$$

$$= \frac{8.5\ \text{KN}/m^3}{9.81\ \text{KN}/m^3} = 0.866$$

We know $W = M \cdot g$

$$\text{Mass} = \frac{W}{g}$$

$$= \frac{25.5}{9.81}$$

$$= 2.593\ \text{kg}$$

$$\text{Mass density } (\rho) = \frac{M}{V} = \frac{2.593}{3}$$

$$= 0.864\ \text{kg}/m^3$$

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## Compressibility of water :-

> The compressibility of a liquid may be defind as the variation in its volume, with the variation on of pressure.

> The variation in the volume of water, with the variation of pressure is so small that all practical purposes so it is neglected.

> Thus the water is to be considered as an incompressible fluid.

## Surface Tension of water :-

> The surface tension of a liquid is its property, which enables it to resist tensile stresses.

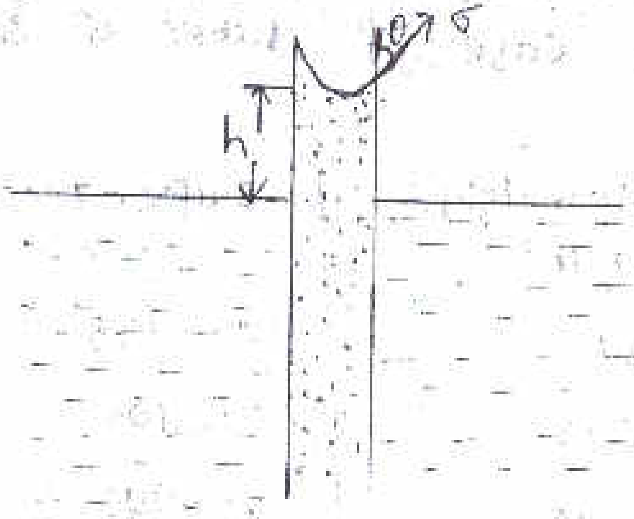
> it is due to the cohesion between the molecules at the surface of a liquid.

> The effect of surface tension may be easily in the case of tubes of smaller diameter.

| Liquid | specific weight (KN/m³) | Surface Tension N/m |
|---|---|---|
| Water | 9.81 | 0.0735 |
| Mercury | 132.8 | 0.5100 |
| Glycerine | 12.45 | 0.0490 |
| Kerosene | 7.85 | 0.0235 |
| Castor oil | 9.41 | 0.0392 |
| Ethyl alcohol | 7.73 | 0.0216 |

## Capillarity of water :-

- When a tube of smaller diameter is dipped in water, the water rises up in the tube with an upward concave surface.
- This is due to the reason that the adhesion between the tube and water molecules is more than the cohesion between the water molecules.
- But when the same tube is dipped in mercury, the mercury depresses down in the tube with an upward convex surface.
- This is due to the reason that the adhesion between the tube & mercury molecules is less than the cohesion between the water molecules.




(Effect of capillarity)

w(mercury) = 132.8

The phenomena of rising water in the tube of smaller diameter is called the capillary rise.

Let $h$ = Height of capillary rise

$d$ = Diameter of capillary tube

$\alpha$ = Angle of contact of water surface.

$\sigma$ = force of surface tension per unit length of the periphery of the capillary tube.

$$\boxed{\text{Capillary rise}(h) = \frac{4\sigma\cos\alpha}{wd}}$$

unit = N/m

1a Calculate the capillary effect in millimeters in a glass tube of 4 m.m diameter when immersed in water, the value of surface tension for water in contact with air, are 0.0735 N/m. The contact angle for water $\theta = 0^\circ$

$1m = 100 cm = 1000 m.m.$

Given data :-

diameter of tube $(d)$ = 4 m.m = $4\times10^{-3}$ m.

Surface Tension of water $(\sigma)$ = 0.0735 N/m

Contact Angle $(\alpha) = 0^\circ$

$$\text{capillary rise }(h) = \frac{4\sigma\cos\theta}{wd}$$

$$= \frac{4\times0.0735\times\cos0^\circ}{9.81\times10^3\times4\times10^{-3}} = 7.5\times10^{-3}$$

$= 7.5$ m.m

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1Q Calculate the effect of capillary rise if the tube is mersed in mercury having surface tension 0.5100 N/m contact angle for mercury $\theta = 130°$

Ans:
Given data:-

dia. of tube $(d) = 4 mm = 4\times10^{-3} m$.

surface Tension for mercury

$(\sigma_m) = 0.5100 N/m$

Contanct angle $(\alpha) = 130°$

$w$ mercury $= 132.8 kN/m^2$

capillary rise for mercury ($h_{mercury}$).

$$\frac{4\sigma\cos\alpha}{w_m d} = \frac{4\times0.5100\times\cos130°}{132.8\times10^3\times4\times10^{-3}}$$

$= -2.47\times10^{-3} m.m$

$= -2.47 m.m$

$= 2.47 m.m$ (depression)

2Q A 5 m.m diameter glass tube is immersed vertically in water, if contact angle is $5°$, find the capillary rise, take surface Tension for water is 0.074 N/m

Sol^n Given data:-

dia of tube $= 5 mm = 5\times10^{-3} m$.

contact angle $\theta = 5°$

specific weight of water $= 9.81 kN/m^3$

Surface Tension for water ($\sigma_w$) $= 0.074$ N/m

Capillary rise ($h_w$) $= \dfrac{4\sigma_w \cos\theta}{w\, d}$

$$= \frac{4 \times 0.074 \times \cos 5^{\circ}}{9.81 \times 10^{3} \times 5 \times 10^{-3}}$$

$$= 6.011 \times 10^{-3}$$

$$= 6 \text{ m.m}$$

3Q Calculate specific weight, density, specific gravity of on Litre of liquid which weighs 7N

Ans Given data =
weighs = 7N = 7000 N/m

Density = $\dfrac{\text{specific weight}}{\text{acceleration due to gravity}}$

$$= \frac{7000 \text{ N/m}^3}{9.81 \text{ m/sec}^2} = 713.55 \text{ kg/m}^3$$

specific Gravity = $\dfrac{\text{Specific weight of liquid}}{\text{Specific weight of water}}$

$$= \frac{7000 \text{ N/m}}{9.81 \times 10^{3} \text{ N/m}^3}$$

$$= 0.713$$

Mathematically $\tau \propto \frac{du}{dy}$

$$\Rightarrow \boxed{\tau = \mu \frac{du}{dy}}$$

$\mu$ = constant of proportionality and is known as co-efficient of dynamic viscosity (or) simply viscosity.

$\frac{du}{dy}$ = rate of change velocity / Rate of shear strain / velocity gradient.

* unit of viscosity

$$\tau = \mu \frac{du}{dy} \Rightarrow \boxed{\mu = \frac{\tau}{\frac{du}{dy}}}$$

$$\Rightarrow \mu = \frac{\frac{\text{Force}}{\text{area}}}{\frac{\text{change in velocity}}{\text{Change in distance}}} = \frac{\text{Force / area}}{\frac{\left(\frac{\text{Length}}{\text{Time}}\right)}{\text{Length}}}$$

$$\mu = \frac{\text{Force / area}}{\frac{\cancel{\text{Length}}}{\text{time}} \times \frac{1}{\cancel{\text{Length}}}} = \frac{\text{Force / area}}{\frac{1}{\text{time}}}$$

$$\mu = \frac{\text{Force}}{\text{Area}} \times \frac{\text{time}}{1}$$

$$\mu = \frac{\text{Force} \times \text{Time}}{(\text{Length})^2}$$

In S.I unit

$\mu = \frac{NS}{m^2} = \frac{Pas}{m^2}$

M.K.S unit

$\mu = \frac{kgf\ sec}{m^2}$

C.g.s unit

$\mu = \frac{dyne\ sec}{cm^2} = poise$

$1 \frac{N}{m} = 1\ pascal$

$1\ kgf = 9.81\ N$ (f = force)

$1\ N = 10^5\ dyne$

$\frac{1\ kgf\ sec}{m^2} = \frac{9.81\ N \times sec}{m^2}$

$= \frac{9.81 \times 10^5\ dyne \times sec}{100^2\ cm^2}$

$= \frac{9.81 \times 10^5 \times dyne\ sec}{10^4\ cm^2}$

$\frac{1\ kgf\ sec}{m^2} = 98.1\ poise$

$9.81\ poise = 9.81 \frac{NS}{m^2}$

$1\ poise = \frac{9.81\ NS}{98.1\ m^2}$

$\boxed{1\ poise = \frac{1\ NS}{10\ m^2}}$

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$1\ poise = \frac{dyne \cdot sec}{cm^2}$

$= \frac{gm \cdot cm}{sec^2} \times \frac{sec}{cm^2}$ $\left[ 1\ dyne = \frac{gm \cdot cm}{sec^2} \right]$

$= \frac{gm}{sec \cdot cm} = \frac{\frac{1}{1000} kg}{S \frac{1}{100} m}$

$= \frac{1}{1000} \times 100 \frac{kg}{sec/m} = \frac{1}{10} \frac{kg}{Sm}$

$1\ poise = \frac{1}{10} \frac{kg}{}$

$$\boxed{10 \text{ poise} = 1 \frac{kg}{m \, sec}}$$

$1 \text{ centipoise} = \frac{1}{100} \text{ poise}$ ← C.G.S unit

**Kinematic viscosity :-**

It is defined as the ratio between dynamatic viscosity and density of fluid it is expressed as '$\nu$' (nu)

Kinematic vi

$$\boxed{\nu = \frac{\mu}{\rho}}$$

**units :-**

$$\text{unit of } \nu = \frac{\text{unit of } \mu}{\text{unit of } \rho}$$

$$= \frac{\frac{\text{Force} \times \text{Time}}{(\text{Length})^2}}{\frac{\text{Mass}}{(\text{Length})^3}}$$

$$= \frac{\text{Force} \times \text{Time}}{(\text{Length})^2} \times \frac{(\text{Length})^3}{\text{Mass}}$$

$$= \frac{\text{Mass} \times \text{acceleration} \times \text{Time} \times \text{length}}{\text{Mass}}$$

$$= \frac{\text{Length}}{(\text{Time})^2} \times \text{Time} \times \text{Length}$$

$$= \frac{(\text{Length})^2}{\text{Time}} = \frac{m^2}{s}$$

> In M.K.S unit or S.I unit of kinematic viscosity is $= \frac{m^2}{s}$

> C.G.S of kinematic viscosity is $cm^2/sec$ (stoke)

$1 \text{ stoke} = 1\ cm^2/sec = \left(\frac{1}{100}\right)^2 m^2/sec$

$= 10^{-4}\ m^2/sec$

$1 \text{ centi stoke} = \frac{1}{100} \times \text{stoke}$

1Q. A plate 0.025 mm distant from a fixed plates, moves at 60 cm/sec and requireds a force of 2N per unit area i.e. $2N/m^2$ to maintain this speed. Determine the fluid viscosity between the platesh

Given data :-

distance between plates $(dy) = 0.025$ mm

$= 0.025 \times 10^{-3}$ m.

velocity $= u = 60$ cm/sec $= 60 \times 10^{-3}$ m/sec

$= 0.6$ m/sec

Force on upper plate $(F) = 2N/m^2$

Let fluid viscosity is '$\mu$' between the plates.

$\tau = \mu \frac{du}{dy}$

$\Rightarrow 2 = \mu \frac{0.6}{0.025 \times 10^{-3}}$

$\Rightarrow \mu = \frac{2 \times 0.025 \times 10^{-3}}{0.6}$

$\Rightarrow \mu = 8.33 \times 10^{-5}\ NS/m^2 = pa \cdot s$

$= 8.33 \times 10^{-5} \times 10$ poise

$= 8.33 \times 10^{-4}$ poise.

$\boxed{1 \text{ poise} = \frac{1}{10} \frac{NS}{m^2} \quad \Rightarrow 1 \frac{NS}{m^2} = 10 \text{ poise}}$

Q) A flat plate of area $1.5 \times 10^6\ mm^2$ is pulled with a speed of $0.4$ m/sec relative to another plate located at a distance of $0.15$ mm from it find the force & power required to maintain this speed. If the fluid separating them is having viscosity as 1 poise.

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Soln Area of plate (A) $= 1.5 \times 10^6\ mm^2$

$= 1.5 \times 10^6$ mm.mm

$= 1.5 \times 10^6 \times 10^{-3} m \times 10^{-3} m$.

$= 1.5\ m^2$

1m = 1000 mm

$\frac{1\ m}{1000} = \frac{1000}{1000}$

$\boxed{\frac{1}{1000} m = 1\ mm}$

$\frac{1}{10^3} m = \boxed{10^{-3} m = 1 mm}$

$10^{6 + (-3) + (-3)} = 10^{6-3-3} = 10^0 = 1$

Speed of plate relative to another plate $du = 0.4$ m/sec

Distance between plates,

$dy = 0.15$ m.m

$= 0.15 \times 10^{-3}$ m.

viscosity $(\mu) = 1$ poise $= \frac{1}{10} \frac{NS}{m^2}$

shear stress ($\tau$) $= \mu \cdot \frac{du}{dy}$

$\Rightarrow \tau = \frac{1}{10} \times \frac{0.4}{0.15 \times 10^{-3}} = 266.66 \frac{N}{m^2}$

shear force = shear stress x Area

$= 266.66 \times 1.5 = 400$ N

power required to move the plate at the speed 0.4 m/sec $= F \times u$

$= 400 \times 0.4 = 160$ w.

$\boxed{1w = Nm/sec}$

3) Find the kinematics viscosity of an oil having density 981 kg/m3 the shear stress at a point in oil is 0.2452 N/m2 and velocity gradient at that point is 0.2 sec

soln shear stress $= 0.2452$ N/m2

velocity gradient $\left(\frac{du}{dy}\right) = 0.2/sec$

Density ($\rho$) $= 981$ kg/m3

shear stress ($\tau$) $= \mu \cdot \frac{du}{dy}$

$\Rightarrow$ viscosity ($\mu$) $= \frac{\tau}{\frac{du}{dy}} = \frac{0.2452}{0.2}$

$\Rightarrow \mu = 1.226$ Ns/m2

kinematic viscosity ($\eta$) $= \frac{\mu}{\rho}$

$= \frac{1.226 \text{ Ns/m}^2}{981 \text{ kg/m}^3}$

$= 0.125 \times 10^{-2}$ m2/sec

$= 0.125 \times 10^{2}$ cm2/sec

$= 12.8$ stoKS

4Q Determine the specific gravity of a fluid having viscosity 0.05 poise and Kinematics viscosity 0.035 stokes.

Soln viscosity ($\mu$) $= 0.05$ poise $= \frac{0.05}{10} \frac{Ns}{m^2}$

Kinematic viscosity ($\eta$) $= 0.035$ stokes

$= 0.035\, cm^2/sec$

$= 0.035 \times 10^{-4}\, m^2/sec$

We know Kinematic viscosity ($\eta$) $= \frac{viscosity}{density}$

$\Rightarrow \boxed{\eta = \frac{\mu}{\rho}}$

$\Rightarrow 0.0035 \times 10^{-4} = \frac{\frac{0.05}{10}}{\rho}$

$\Rightarrow \rho = \frac{\frac{0.05}{10}}{0.035 \times 10^{-4}} = \frac{0.05}{10} \times \frac{1}{0.035 \times 10^{-4}}$

$= 1428.5\ kg/m^3$

specific gravity of fluid $= \frac{1428.5}{1000}$

$= 1.4285$

5Q Determine the viscosity of liquid having Kinematics viscosity 6 stokes and specific gravity 1.9. Also calculate the density & specific weight of the given liquid.

Soln Given data :-

$\eta = 6$ stokes $= 6\, cm^2/sec = 6 \times 10^{-4}\, m^2/s$

specific gravity of liquid $= 1.9$

let viscosity gravity of liquid $= \mu$

$$\text{S.P. gravity} = \frac{\text{Density of liquid}}{\text{Density of water}}$$

$$\Rightarrow 1.9 = \frac{\text{Density of liquid}}{1000}$$

Density of liquid $= 1000 \times 1.9 = 1900\ kg/m^3$

Now kinematic viscosity $(\eta) = \dfrac{\text{viscosity}}{\text{density}}$

$$\eta = \frac{\mu}{\rho}$$

$$\Rightarrow 6 \times 10^{-4} = \frac{\mu}{1900}$$

$$\Rightarrow \mu = 6 \times 10^{-4} \times 1900 = 1.14\ Ns/m^2$$

$$= 1.14 \times 10 \text{ poise} = 11.40 \text{ poise}$$

$$\boxed{\begin{array}{l} 1 \text{ poise} = \frac{1}{10}\ Ns/m^2 \\ 1\ Ns/m^2 = 10 \text{ poise} \end{array}}$$

8 May 2021

10 Find the kinematic viscosity of an oil having 980 kg/m3 when a certain point in the oil, shear stress is 0.25 N/m2 and velocity gradient 0.3/sec.

Sol Given data :-

velocity gradient $\left(\dfrac{du}{dy}\right) = 0.3/sec$

Shear stress $= 0.25\ N/m^2$

Density $= 980\ kg/m^3$

Shear stress $\tau = \mu \cdot \dfrac{du}{dy}$

$$\Rightarrow 0.25 = \mu \cdot 0.3/sec$$

$$\Rightarrow \mu = \frac{0.25}{0.3} = 0.833\ Ns/m^2$$

Kinematic viscosity ($\eta$) = $\dfrac{\text{dynamic viscosity } (\mu)}{\text{density } (\rho)}$

$\Rightarrow \eta = \dfrac{0.833 \ Ns/m^2}{980 \ kg/m^3}$

$\Rightarrow 8.503 \times 10^{-4} \ m^2/sec$

$\Rightarrow 8.503 \times 10^{-4} \times 10^{4} \ cm^2/sec$

$\Rightarrow 8.50$ stokes.

# Fluid pressure & its measurement

## Fluid pressure / Intensity of pressure :-

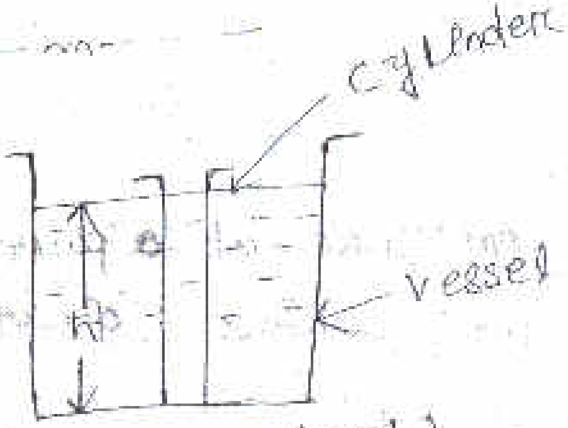
> Whenever a liquid such as oil / water etc. is contained in a vessel, it exerts force at all points on the sides and bottom of the container.

> This force per unit area is called pressure / fluid pressure / Intensity of pressure

$$\boxed{\text{Intensity pressure } (p) = \frac{P}{a}}$$

> The direction of this pressure is always right angles to the surface with which the fluid at rest.

## Pressure Head




(pressure head)

$$p = \frac{\text{weight of liquid in the cylinder}}{\text{Area of cylinder base}}$$

$$= \frac{w \cdot h \cdot A}{A} = w \cdot h$$

$$\Rightarrow \boxed{p = w \cdot h}$$

where $w$ = specific weight of liquid

$h$ = Height of liquid in the cylinder

$A$ = Area of cylinder base.

$$w = \frac{W}{A \cdot h}$$

$$W = w \cdot h$$

~~this~~ ~~equa~~ From $\boxed{p = wh}$, it shows that the intensity of pressure ~~at~~ any point in a liquid is proportional to its depth from the surface ($w$ is constant)

unit :-

① N/m² or KN/m²

② As the height of equivalent liquid column.

31 may 2021

Q1 find the pressure at a point 4m. below the free surface of water.

Ans

pressure $= w \cdot h$

$= 9.81 \times 4$

$= 39.24$ KN/m²

2Q A steel plate is immersed in an oil of specific weight 7.5 KN/m³ up to a depth of 2.5m. what is the intensity of pressure on the plate

Ans specific weight of oil (w) = 7.5 kN/m³

depth (h) = 2.5 m.

pressure intensity on the plate (p) = wh

$= 7.5 \times 2.5$

$= 18.75\ kN/m^2$ (or) kPa

3Q calculate the height of water column equivalent to a pressure of 0.15 MPa

Ans pressure = 0.15 MPa

$= 0.15 \times 10^6$ pa $= 0.15 \times 10^3 \times 10^3$ pa

$= 0.15 \times 10^3$ kPa

pressure = $w \times h$

$\Rightarrow 0.15 \times 10^3\ kPa = 9.81\ kN/m^3 \times h$

$\Rightarrow h = \dfrac{0.15 \times 10^3}{9.81} = 15.29 \simeq 15.3\ m.$

4Q what is the height of an oil colum of specific gravity 0.9 equivalent to a pressure of 20.3 kPa

Ans specific gravity (g) = 0.9

pressure = 20.3 kPa

Height of oil column (metre) = ?

(sp) = $\dfrac{\text{specific weight of oil}}{\text{specific wt of water}} = 0.9$

specific weight of oil $= 0.9 \times$ specific weight of water

$= 0.9 \times 9.81$ KN/m3

$\Rightarrow$ $w_{oil} = 8.829$ KN/m$^3$

we know pressure intensity $= w_{oil} \times h$

$\Rightarrow 20.3$ kpa $= 8.829$ KN/m$^3 \times h$

$$\Rightarrow h = \frac{20.3}{8.829} = 2.3 \text{ m}$$

H.W

5Q Find the pressure at a point 1.6 m. below the free surface of water in a swimming pool.

Sol$^n$ $P = w \cdot h$

$= 9.81 \times 1.6$

$= 15.696$ KNm2 / kpa

6Q A point is located at a depth of 1.6 m. from the free surface of an oil of specific weight 8.0 KN/m$^3$ calculate the intensity of pressure at the point.

Sol$^n$ $h = 1.6$ m.

$w = 8.0$ KN/m3

$(p) = wh = 8.0 \times 1.6 = 12.8$ Kpa

3Q Find the height of water column corresponding to a pressure of 5.6kpa

Sol$^n$ pressure (P) = 5.6 kpa

$p = w \times h$

$h = \frac{P}{W} = \frac{5.6}{9.81} = 0.57m.$

4Q Determine the height of an oil column of specific gravity 0.8, which will cause of 25 kpa. pressure

Sol$^n$ specific gravity = 0.8

pressure = 25 kpa

specific gravity = $\frac{\text{specific weight of oil}}{\text{specific weight of water}}$

$w_{oil} = 0.8 \times 9.81$

$= 7.848 \ kN/m^3$

$p = w_{oil} \times h$

$\Rightarrow 25kpa = 7.848 \times h$

$h = \frac{25kpa}{7.848 kN/m^3} = 3.185 \ m.$

50 Calculate the height of mercury column equivalent to a gauge pressure of 150kpa.

Sol^n

$p = \omega \times h$

$150kpa = \omega \times h$

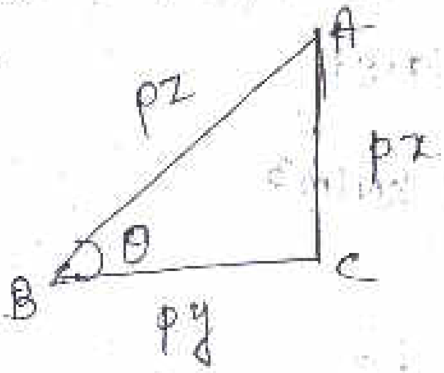
$150kpa = 132.8\ kN/m^3 \times h$

$h = \frac{150kpa}{132.8\ kN/m^3} = 1.129m.$

## Pascal's Law

It states, "the intensity of pressure at any point in a fluid at rest is same in all directions".

proof :-




Consider a very small Right angle of triangular element ABC of liquid.

Let $p_x$ = intensity of horizontal pressure on the element of liquid.

$p_y$ = Intensity of vertical pressure on the element of liquid.

$p_z$ = Intensity of pressure on the diagonal of the triangular element of liquid.

1. June 2021

$\theta$ = Angle of triangular element of the liquid.

pressure on vertical side AC of the liquid

$P_x = p_x \times AC$ —— (I)

pressure on horizontal side BC of the liquid

$P_y = p_y \times BC$ —— (II)

pressure on diagonal AB of the liquid

$P_z = p_z \times AB$ —— (III)

since the element of liquid is at rest therefore the sum of horizontal and vertical component of the liquid pressure must be equal to zero. Resolving the forces horizontally

$$P_z \sin\theta = P_x \quad \left[\begin{array}{l} P_z = p_z \cdot AB \\ P_x = p_x \cdot AC \end{array}\right]$$

$\Rightarrow p_z AB \sin\theta = p_x \cdot AC$

from the geometry of figure

$AB \sin\theta = AC$

$\Rightarrow p_z \cdot AC = p_x \cdot AC$

$\Rightarrow p_z = p_x$ ——— (iv)

Resolving forces vertically, we get

$p_z \cdot \cos\theta = p_y$

$\Rightarrow p_z \cdot AB \cos\theta = p_y \cdot BC$

$\Rightarrow$

from the geometry

$AB \cos\theta = AC$

$\Rightarrow p_z \cdot BC = p_y \cdot BC$

$\Rightarrow p_z = p_y$ ——— (v)

from eq$^n$ 4 & 5 we get

$$\boxed{p_x = p_y = p_z}$$

i.e the intensity of pressure at any point in a fluid is same in all direction

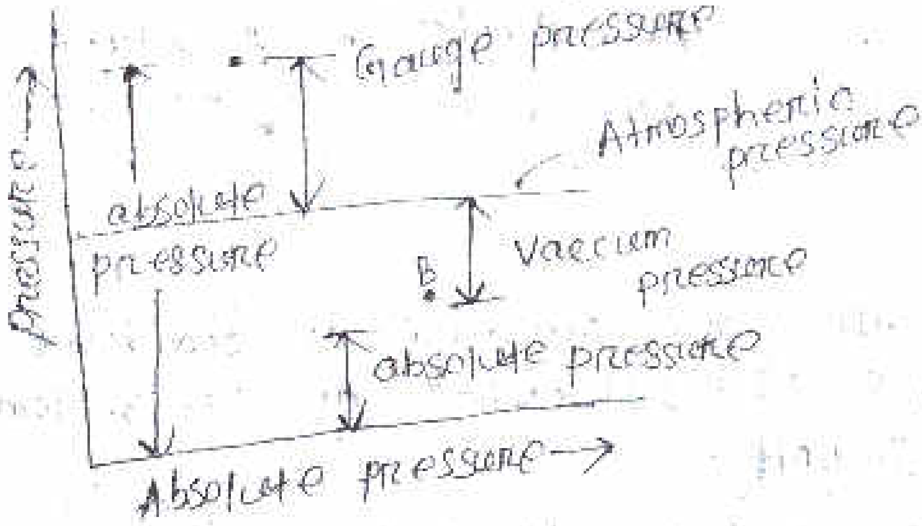
2 June, 2021

The pressure on a fluid is measured in two different system.

$\Rightarrow$ In one system it is measured above the absolute zero or complete vaccum and it is called absolute pressure

$\Rightarrow$ and in other system pressure is measured above atmospheric pressure is called gauge pressure.

$$\boxed{p = \rho \cdot g \cdot h}$$ $\rightarrow$ Hydrostatic Law




Absolute pressure :- It is defined as the pressure which is measured with reference to absolute vaccum pressure.

Gauge pressure :- It is defind as the pressure measured with the help of pressure measuring instrument in which atmospheric pressure is taken as datum. The atmospheric pressure on that scale is zero.

Vaccume pressure It is defined as the pressure below the atmospheric pressure.

Absolute pressure = Atmospheric pressure + Gauge pressure

$$P_{ab} = P_{atm} + P_g$$

$$\text{vaccum pressure} = \text{Atmospheric pressure} - \text{Absolute pressure}$$

NOTE

① The atmospheric pressure at sea level at $15^0 C$ is $103 \cdot 1\ kN/m^2$ or $10 \cdot 13 N/cm^2$ in S.I unit.

In case of MKS unit it is equal to $1 \cdot 033\ kgf/cm^2$

② The atmospheric pressure head is $760\ m \cdot m$ of mercury or $10 \cdot 33\ m$ of water.

**1Q** What are the gauge pressure and absolute pressure at a point 3m below the free surface of liquid having density of $1 \cdot 53 \times 10^3\ kg/m^3$. if atmospheric pressure is equivalent to $750\ m \cdot m$ of mercury The specific gravity of mercury is $13 \cdot 6$ and density of water $= 1000\ kg/m^3$

**Sol**

$$\text{specific gravity} = \frac{\text{specific weight of mercury}}{\text{specific weight of water}}$$

$$w = \rho \cdot g \quad = \frac{\text{density of mercury} \times g}{\text{density of water} \times g}$$

$$= \frac{\text{density of mercury}}{\text{density of water}}$$

$\Rightarrow 13.6 = \frac{\rho m}{1000}$

density of mercury $= 13.6 \times 1000$

$= 13600 \ kg/m^3$

Atmospheric pressure ($p_0$) $= \rho g h$

$= 13600 \times 9.81 \times 0.75$

$= 100062 \ N/m^2$

$1 kN = 10^3 N$

$= \frac{100062 \times 10^3 N/m^2}{10^3} = 100062 \times kg/m^2$

$= 100.062 \times kN/m^2$

pressure at point which is at depth of 3m. from the free surface of liquid

$p_g = \rho g h$

$= 1.53 \times 10^3 \times 9.81 \times 3$

$= 45027.9$

$= 45028$

Absolute pressure = Gauge pressure + atmospheric pressure

$= 45028 + 100062$

$= 145090 \ N/m^2$

$= 145.090 \ kN/m^2$

3 June 2021 OMM SAI RAM

## Measurement of pressure :-

The pressure of a fluid is measured by following devices.

(i) Manometers

(ii) Mechanical Gauge

### Manometers :-

It is defined as the devices used for measuring pressure at a point in a fluid by balancing the column of fluid by the same or another column of fluid.

This is two type:-

(i) simple Manometer

(ii) Differential Manometer

### Mechanical Gauge

These devices are used for measuring the pressure by balancing the fluid column by the spring on dead weight.

### (i) simple manometer

It consist of glass tube having one of its ends connected to a point where pressure is to be measured and the other end remains open to atmosphere

Common type of simple manometers are :-

(a) piezometer

(b) U-tube manometer

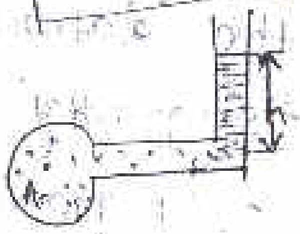
(c) single column manometer.

## piezometer :-

> It is a simplest form of manometer used for measuring gauge pressure one end of this manometer is connected to be point where pressure is to be measured and other end is open to atmosphere.

> The rise of liquid gives the pressure head that point.

> If a point 'A' the hight of liquid (h) in piezo meter tube, then the pressure at A is $\boxed{P_A = \rho g h}$

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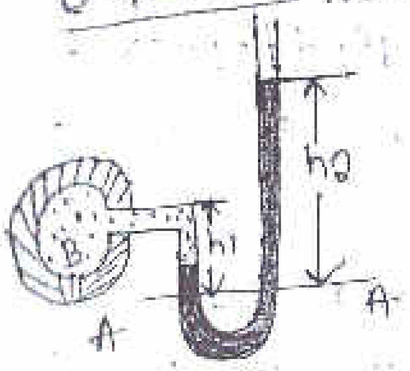
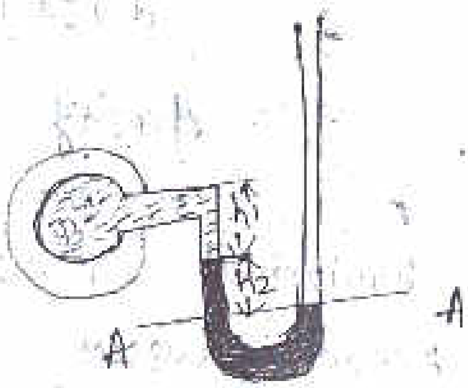
## U-tube manometer :-




(for gauge pressure)




(for vacuum pressure)

> It consists of glass tube bent in u-shape one end of which is connected to a point at which pressure is to be measured and other end remains open to the atmosphere.

> The tube generally contains mercury or any other liquid whose specific gravity is greater than the sp gravity of the liquid whose pressure is to be pressure.

For Gauge Pressure :-

Let B is the point at which pressure whose value is 'p' is to be measured

Datum line is A - A

Let $h_1$ = Height light liquid above the datum line

$h_2$ = Height of heavy liquid above the datum line.

$S_1$ = sp gravity of lighter liquid.

$S_2$ = sp gravity of heavy liquid.

$\rho_1$ = density of lighter liquid = $1000 \times S_1$

$\rho_2$ = density of heavier liquid = $1000 \times S_2$

As the pressure is the same for the horizontal surface.

Hence pressure above horizontal datum line A-A in the left column of u-tube manometer is same pressure above A A in left column

$= p + \rho_1 g h_1$

pressure above A-A in the right column

$= \rho_2 g h_2$

Hence equation two pressure

pressure in leftside column = pressure in right side of column

$\Rightarrow p + \rho_1 g h_1 = \rho_2 g h_2$

$\Rightarrow \boxed{p = \rho_2 g h_2 - \rho_1 g h_1}$

For vaccum pressure:—

pressure above A-A in the left side column $= \rho_2 g h_2 + \rho_1 g h_1 + p$

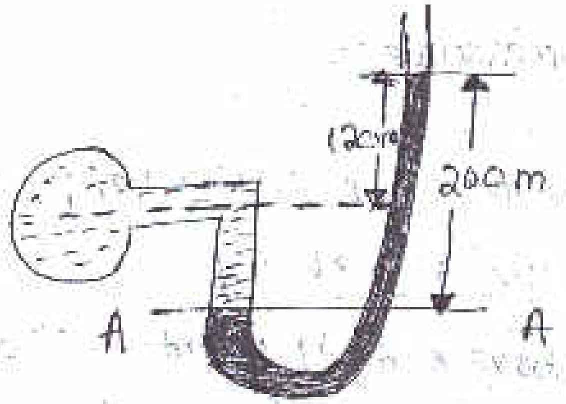
pressure above A-A is right side of column = 0

pressure in left column = pressure in right column

$= \rho_2 g h_2 + \rho_1 g h_1 + p = 0$

$\boxed{p = -(\rho_2 g h_2 + \rho_1 g h_1)}$

16 The right limb of simple u-tube manometer containing mercury is open to atmosphere while the left limb is connect ~~atmosphere while the left limb~~ to a pipe in which a fluid of sp. gravity 0.9 is flowing. The centre of pipe is 12cm below the level of mercury in the right limb find pressure of liquid if the difference of mercury level in the two limbs is 20cm.

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Soln Given data:—

specific gravity of fluid = 0.9 ($S_1$)
(left limb)

density of fluid ($\rho_1$) = $1000 \times S_1 = 0.9 \times 1000$
= 900 kg/m3

specific gravity of mercury in right limb ($S_2$) = 13.6

density of mercury ($\rho_2$) = $13.6 \times 1000$
= 13600 kg/m3

Difference of mercury level $(h_1)$ = 20cm - 12cm

= 8cm = 0.08m.

$h_2$ = 20cm = 0.2m.

Let p = pressure of fluid in pipe at datum line. ∴ pressure in left limb = pressure in right limb

$\Rightarrow p + \rho_1 g h_1 = \rho_2 g h_2$

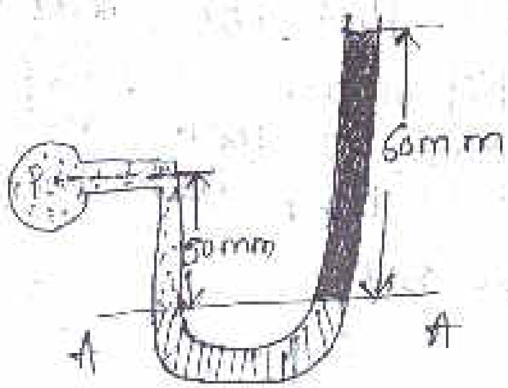
$\Rightarrow p = \rho_2 g h_2 - \rho_1 g h_1 = g(\rho_2 h_2 - \rho_1 h_1)$

$\Rightarrow p = 9.81(13600 \times 0.2 - 900 \times 0.08)$

$\Rightarrow p = 25976.88 \ N/m^2$

$= 25.976 \ kN/m^2 / kPa$

2) A simple manometer containing mercury is used to measure the pressure of water in a pipeline. The mercury level in the open tube is 60mm higher than that on the left tube. If height of water in the left tube is 50mm find pressure in the pipe in terms of head of water.




$p = \rho g h$

Pressure Head $= \frac{p}{\rho g}$

Given data :-

height of water in left limb ($h_1$) = 50 mm

sp gravity of water ($S_1$) = 1.0

height of mercury in right limb ($h_2$) = 60 mm

sp gravity of mercury ($S_2$) = 13.6

Let H = pressure in the pipe in terms of head of water at datum pressure head is equal in Left & Right limb.

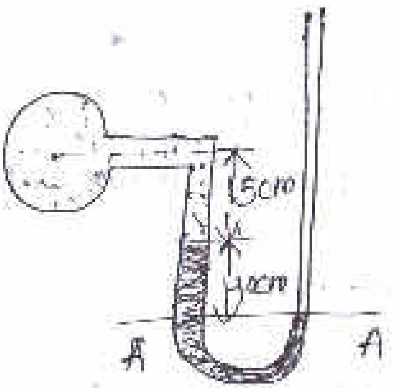
$\Rightarrow H + S_1 \times h_1 = S_2 h_2$

$\Rightarrow H + 1.0 \times 50 = 13.6 \times 60 mm$

$\Rightarrow H = 13.6 \times 60 - 1.0 \times 50 m.m$

$\Rightarrow H = 816 - 50 = 766 mm$ of water

3Q A simple u-tube manometer containing mercury is connected to a pipe in which a sp. gravity 0.8 and having vaccume pressure is flowing. The other end of manometer is open to atmospher. find the vaccume pressure in pipe, if difference of mercury level in the two limbs is 40 cm and the height of fluid in the left from the centre of pipe is 15 cm below.




$S_1 = 0.8$

$S_2 = 13.6$

$\rho_1 = 1000 \times 0.8 = 800\, kg/m^3$

$\rho_2 = 13600\, kg/m^3$

Difference in mercury level $h_2 = 0.4\, m$

$h_1 = 0.15\, m$.

Let pressure in pipe $= p$.

pressure above the datum on two side should be equal i.e

$p + \rho_1 g h_1 + \rho_2 g h_2 = 0$

$\Rightarrow p + 800 \times 9.81 \times 0.15 + 13600 \times 9.81 \times 0.4 = 0$

$\Rightarrow p + 1177.2 + 53366.4 = 0$

$\Rightarrow p + 54543.6 = 0$

$p = -54543.6\, N/m^2$

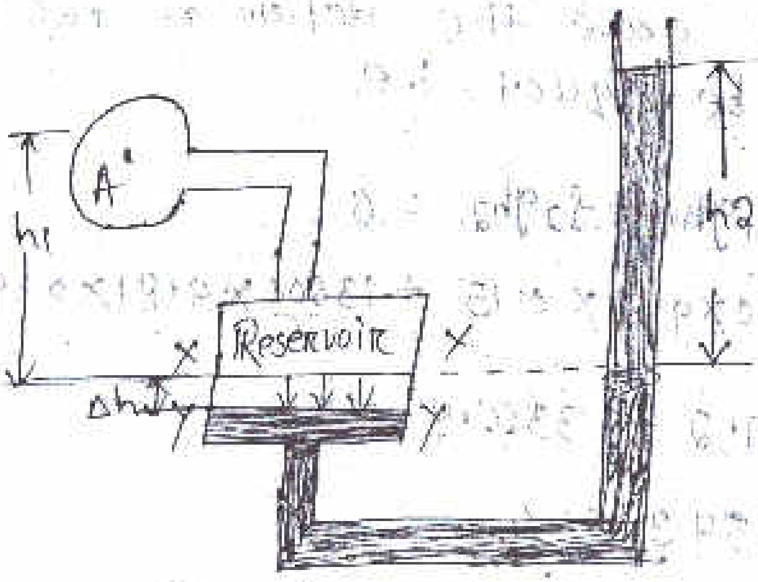
(2) Single column manometer

① vertical column manometer
② Inclined single column manometer

vertical column manometer

Let x - x be the datum line in the reservoir and in the right limb of manometer, when it is not connected to the pipe.

when the manometer is connected to the pipe, due to high pressure at A, the heavy liquid in the reservoir will be pushed downward and will rise in the right limb.




( vertical single column manometer )

Let $\Delta h$ = fall of heavy liquid in the Reservoir
$h_2$ = Rise of heavy liquid in right limb
$h_1$ = height of centre of pipe above X-X

$pA$ = pressure at A (which is to be measured)

$A$ = cross-sectional area of Reservoir

$a$ = area of right limb

$S_1$ = sp gravity of liquid in pipe

$S_2$ = s.p gravity of heavy liquid in reservoir and right limb.

$\rho_1$ = density of liquid in pipe

$\rho_2$ = density of liquid in reservoir

fall of heavy liquid in reservoir will cause rise of heavy liquid in right limb.

$A \times \Delta h = a \times h_2$

$$\boxed{\Delta h = \frac{a \times h_2}{A}}$$

Pressure in the right limb above Y-Y

$= \rho_2 \times g \times (\Delta h + h_2)$

pressure in the left limb above Y-Y

$= \rho_1 \times g \times (\Delta h \times h_1) \times pA$

pressure in should be equal

$\rho_2 g \times (\Delta h + h_2) = \rho_1 \times g \times (\Delta h + h_1) + pA$

$\Rightarrow pA = \rho_2 g \times (\Delta h + h_2) - \rho_1 g \times (\Delta h + h_1)$

$\Rightarrow P_A = \Delta h(\rho_2 g - \rho_1 g) + g(h_2\rho_2 - h_1\rho_1)$

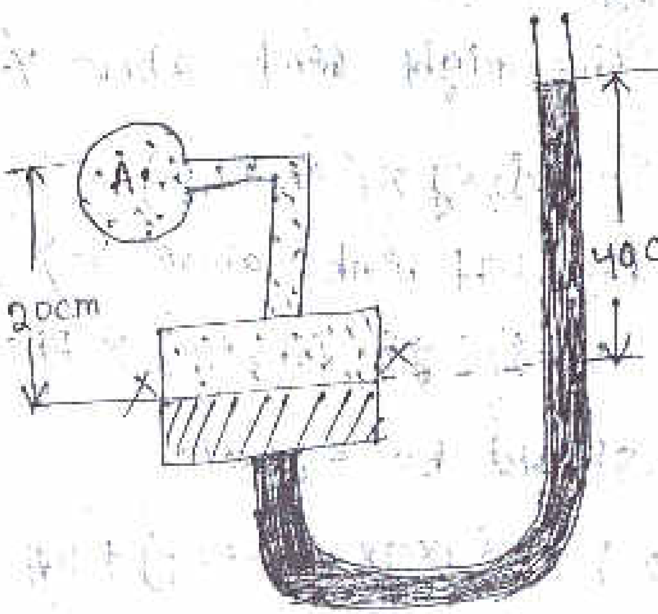
$\Rightarrow P_A = \frac{a \times h_2}{A}(\rho_2 g - \rho_1 g) + h_2\rho_2 g - h_1\rho_1 g$

Hence $\frac{a}{A}$ becomes very small so ignore then eqn becomes.

$$\boxed{P_A = h_2\rho_2 g - h_1\rho_1 g}$$

8 June 2021

**Q1** A single column manometer is connected to a pipe containing liquid of sp. gravity 0.9. Find the pressure in the pipe if area of reservoir is 100 times that of area of the tube for the manometer as shown. The sp gravity of mercury is 13.6.



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$\underline{\underline{sol^n}}$

Given data:-

specific gravity of fluid in pipe $S_1 = 0.9$

Density of fluid in pipe $\rho_1 = 0.9 \times 1000 = 900\,kg/m^3$

specific gravity of heavy liquid $= 13.6$ ($S_2$)

Density of heavy liquid ($\rho_2$) $= 13600\,kg/m^3$

Area of Reservoir $= 100 \times$ area of right limb of tube

$\Rightarrow A = 100 \times a$

$\Rightarrow \frac{A}{a} = 100$

Height of liquid ($h_1$) $= 20\,cm = 0.2\,m$.

Rise of mercury in right limb ($h_2$) $= 40\,cm = 0.4\,m$.

PA = pressure of pipe which is to be measured

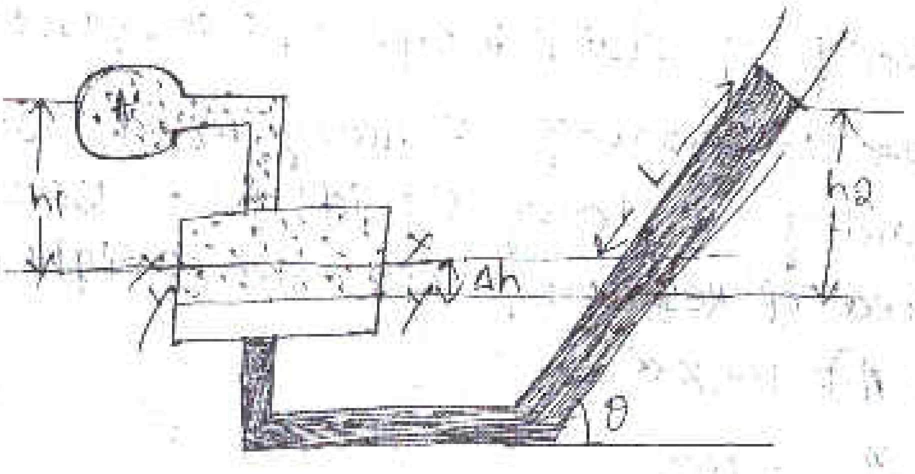
$$P_A = \frac{a}{A} h_2 [\rho_2 g - \rho_1 g] + h_2 \rho_2 g - h_1 \rho_1 g$$

$$= \frac{1}{100} \times 0.4 (13600 \times 9.81 - 900 \times 9.81) + 0.4 \times 13600 \times 9.81 - 0.2 \times 900 \times 9.81$$

$$P_A = \frac{0.4}{100} [133416 - 8829] + 53366.4 - 1765.8$$

$$\Rightarrow P_A = 52098.94 \approx 5.21\,N/cm^2$$

# Inclined single column manometer :-




→ It is a modified from of a U-tube manometer in which a reservoir, having cross-sectional area (about 100 times) as compared to the area of tube connected to one of it's limb (say left limb) of the manometer.

→ Due to inclination the distance moved by the heavy liquid in the right limb will be more.

Let L = Length of heavy liquid moved in the right limb x-x

θ = inclination of right limb with horizontal

$h_2$ = vertical rise of heavy liquid in right limb from x-x = $L \times \sin\theta$

Pressure at A is $P_A = h_2 \rho_2 g - h_1 \rho_1 g$

$\Rightarrow$ $\boxed{P_A = L\sin\theta \rho_2 g - h_1 \rho_1 g}$

Differential manometer or U tube manometer

> These are the devices used for measuring the difference of pressures between two points or in two different pipes.

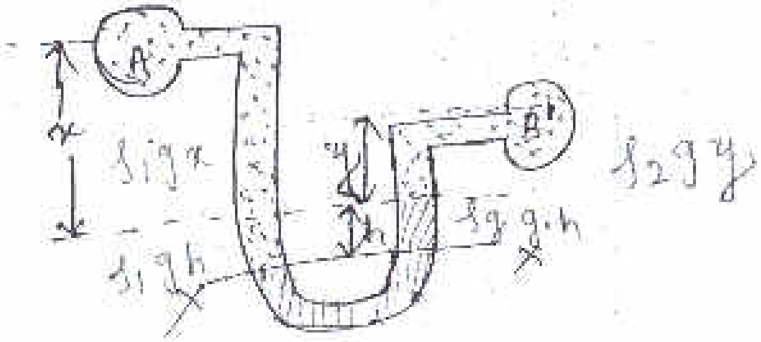
> A differential manometer consists of a U-tube containing heavy liquid, whose two ends are connected to the point, whose difference of pressure is to be measured.

differential manometer are of 2 types

① - U-tube differential manometer

② Inverted

U-tube differential manometer 10 June 2021




Two points at different levels

Let the two points A and B are at different level and also contains liquid of different specific gravity.

These points are connected to the U-tube differential manometer.

Let pressure at A and B are PA & PB respectively.

Let h = difference of mercury level in the U-tube

y = difference of centre of B from the mercury level in right limb.

x = distance of centre of A from the mercury level in left limb.

$\rho_1$ = Density of liquid at A

$\rho_2$ = Density of liquid at B

$\rho_g$ = Density of heavy liquid or mercury.

Taking datum line as X-X

pressure above X-X in the left limb

$= PA + \rho_1 g x + \rho_1 g h$

$= PA + \rho_1 g (h+x)$ —— ①

pressure above X-X in the right limb

$= \rho_g \times g \times h + \rho_2 g y + PB$ —— ②

at datum eqn ① = eqn ②

$\rho_1 g (h+x) + PA = \rho_2 g h + \rho_2 g \cdot y + PB$

$\Rightarrow PA - PB = \rho_g g h + \rho_2 g y + \rho_1 g (h+x)$

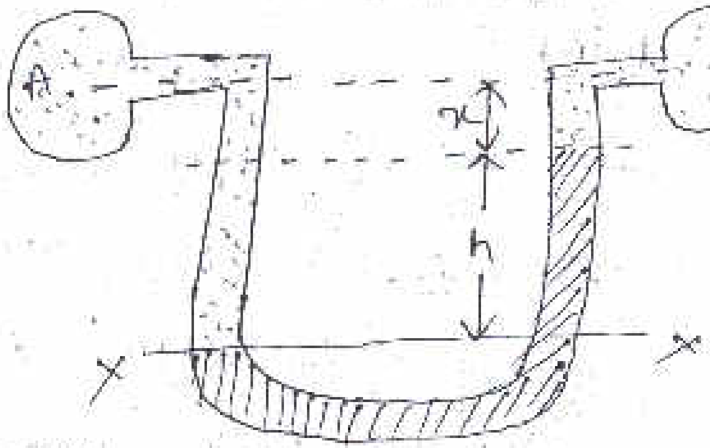
$\Rightarrow p_A - p_B = \rho_g \cdot g \cdot h + \rho_2 g y - \rho_1 g h - \rho_1 g x$

$\Rightarrow p_A - p_B = h \cdot g \, (\rho_g - \rho_1) + \rho_2 g y - \rho_1 g x$

Difference at pressure A and B = $p_A - p_B$

$= h \cdot g \, (\rho_g - \rho_1) + \rho_2 g y - \rho_1 g x$

when two pipe are at same level :-

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A and B are at same level and contai same liquid of density $\rho_1$ Then pressure above datum (x - x) in the

left limb = $\rho_1 g h + \rho_1 g x + p_A$

pressure above datum (x - x) in the right limb = $\rho_g \cdot g \cdot h + \rho_1 g x + p_B$

at datum line pressure at right limb = pressure at left limb.

$p_B + \rho_g \cdot g h + \rho_1 \cdot g x = p_A + \rho_1 g h + \rho_1 g x$

$\Rightarrow p_A - p_B = \rho_1 g h - \rho_g \cdot g \cdot h$

$$\boxed{p_A - p_B = g \cdot h_1 (\rho_1 - \rho_g)}$$

19. A pipe contains an oil of specific gravity 0.9. A differential manometer connected at the two points at same level A and B shows a difference in mercury level is 15cm. find the difference of pressure at the two point.

Soln

specific gravity oil $= 0.9$

density of oil, $\rho_1 = 0.9 \times 1000 = 900\ kg/m^3$

density of heavy liquid $(\rho_g) = 13600\ kg/m^3$

difference in mercury level $=$ 15cm $(h) = 0.15m$

when pipes are at same level pressure difference $p_A - p_B = (\rho_g - \rho_1)\, g h$

$= (13600 - 900) \times 9.81 \times 0.15$

$= 18688.05\ N/m^2$

$= 18.688\ kPa$

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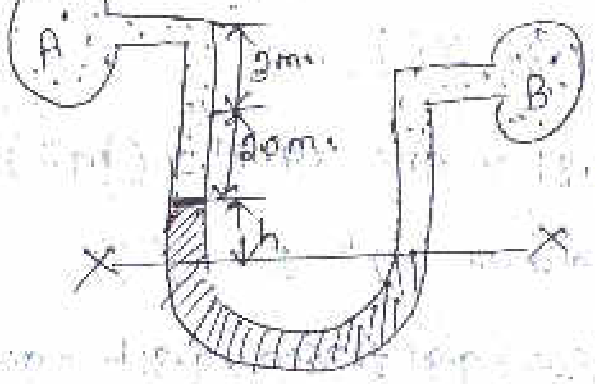
Q. A differential manometer is connected at the two points A and B of two pipes as shown in fig. The pipe A contains the liquid of specific gravity 1.5 while pipe B contains a liquid of specific gravity 0.9. The pressure at A & B are 1 kgf/cm² and 1.80 kgf/cm² respectively. Find the difference in mercury level in the differential manometer.

$1 kgf = 9.81 N$




soln: Given data :-

specific gravity of liquid at A, $S_1 = 1.5$

density of liquid at A, $\rho_1 = 1500 kg/m^3$

specific gravity of liquid at B, $S_2 = 0.9$

density of liquid at B, $\rho_2 = 900 kg/m^3$

pressure at A $P_A = 1 kgf/cm^3$

$= 9.81 N/cm^2$

$= 9.81 \times 10^4 N/m^2$

pressure at B, $P_B = 1.8$ kgf/cm$^2$
$= 1.80 \times 9.81 \times 10^4$ N/m$^2$

$1 cm = 10^{-2} m$.
$1 cm^2 = (10^{-2})\, m^2$
$= 10^{-4} m^2$

Density of mercury $\rho_m = 13600$ kg/m$^3$

Taking x-x as datum line. pressure above x-x in the left limb is

$= P_A + 1500 \times 9.81 \times (2+3) + 13.6 \times 1000 \times 9.81 \times h$

$= 9.81 \times 10^4 + 7500 \times 9.81 + 13600 \times 9.81 \times h$ ——— (i)

pressure above x-x in right limb is

$P_B + 900 \times 9.81 \times (h+2)$

$= 1.8 \times 10^4 \times 9.81 \times 900 \times 9.81 \times (h+2)$ ——— (ii)

we know at datum eqn ① = eqn ②

$\Rightarrow 9.81 \times 10^4 + 7500 \times 9.81 + 13600 \times 9.81 h = 900 \times 9.81 (h+2) + 1.8 \times 10^4 \times 9.81$

Dividing each by $1000 \times 9.81$ we get

$13.6h + 7.5 + 10 = (h+2) \times 0.9 + 18$

$\Rightarrow 13.6h + 17.5 = 0.9h + 1.8 + 18$

$\Rightarrow h(13.6 - 0.9) = 19.8 - 17.5$

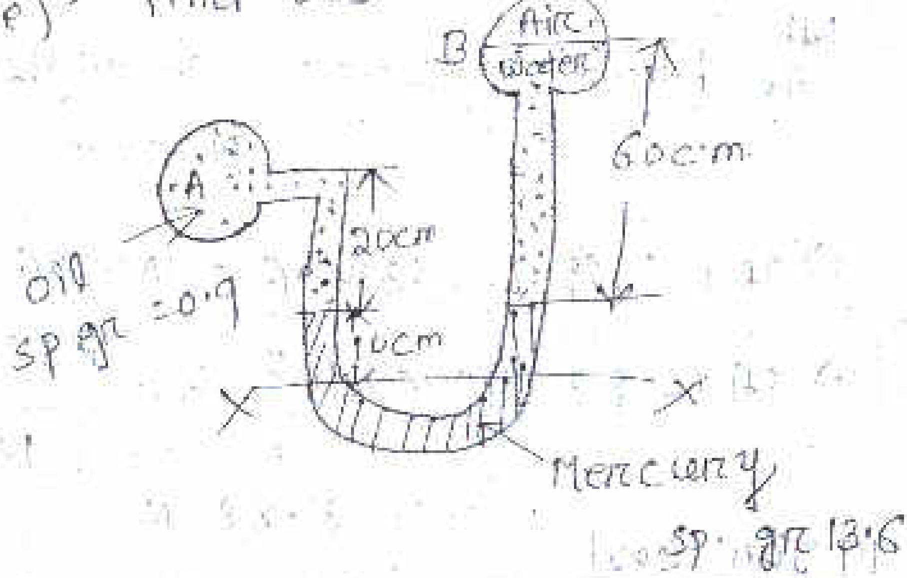
$\Rightarrow h \times 12.7 = 2.3$

$h = \frac{2.3}{12.7} = 0.181$ m. $\simeq 18.1$ cm

Ans

18 June 2021 O.S.M

**1Q** A differential manometer is connected at the two points A and B as shown in the figure. At B air pressure is 9.81 N/cm2 (absolute). Find absolute pressure at A.




**Soln** Given data

Air pressure at B = 9.81 N/cm$^2$

$\Rightarrow P_B = 9.81 \times 10^4$ N/m$^2$

Density of oil = 900 kg/m$^3$

Density of mercury = 13600 kg/m$^3$

Let pressure at A is $P_A$

Taking datum line of X-X

pressure above X-X in the right limb

$= P_B + 1000 \times 9.81 \times 0.6$

$= 9.81 \times 10^4 + 1000 \times 9.81 \times 0.6$

$= 103986$ N/m$^2$ —— ①

Pressure above x-x in left limb

$= P_A + 900 \times 9.81 \times 0.2 + 13600 \times 9.81 \times 0.1$ N

$= P_A + 1765.8 + 13341.6$ —— (i)

At datum pressure at right limb = pressure left limb

$\Rightarrow P_A + 1765.8 + 13347.6 = 103986$

$\Rightarrow P_A = 88878.6\ N/m^2 = \dfrac{88878.6}{10^4}$

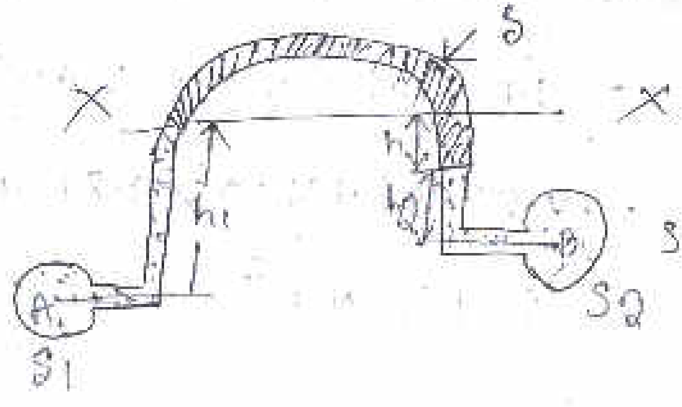
$= 8.88$ N

19 Jan 2021

## Inverted U-Tube differential manometer

It consists of an inverted U-tube containing a light liquid. The two ends of the tube are connected to the points whose difference of pressure is to be measured.

→ It is used for measuring difference in low pressure.



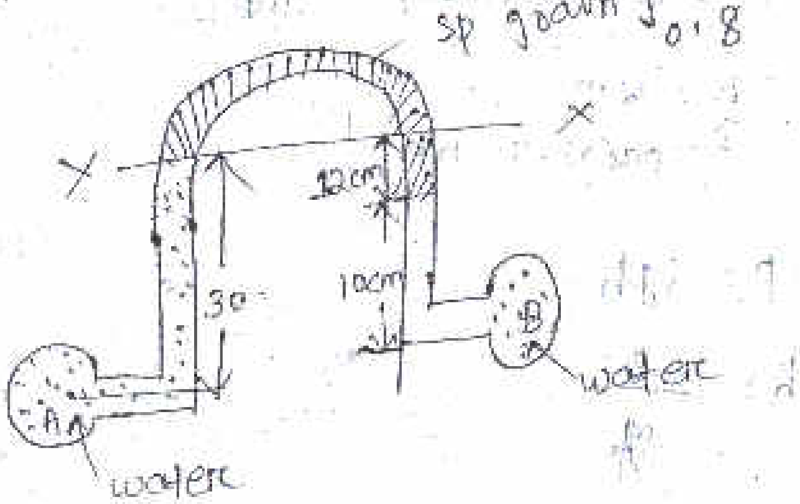

S.P gravity

Let an inverted u-tube manometer connected to the two points A and B let pressure at A is more than pressure at B consider.

$h_1$ = hight of liquid in left limb below the datum x-x

$h_2$ = hight of liquid in right limb

<u>1a</u> water is flowing thorough two different pipes to which inverted differential manometer having an oil as specific gravity 0.8 is connected. The pressure in pipe A is 5.25KN/m² . find pressure at B for the differential manometer.




Given data :-

Pressure at A = 5.25KN/m²

Pressure below x-x in left limb

= PA - 1000 × 9.81 × 0.3 = PA - 2943

= 5.25×10³ - 2943 = 2307N/m² ——— (1)

Pressure below x-x in right limb

$P_B - 1000 \times 9.81 \times 0.1 - 800 \times 9.81 \times 0.12$

$= P_B - 981 - 941.76 = P_B - 1922.76$ —— (ii)

equating eq$^n$ (i) & eq$^n$ (ii)

$P_B - 1922.76 = 2307$

$\Rightarrow P_B = 2307 + 1922.76$

$$\boxed{P_B = 4229.76 \ N/m^2}$$

$P_B = \frac{4229.76}{1000} \ kN/m^2 = 4.229 \ kN/m^2$

Pressure head in pipe in 2m of water

$h = 2m$
pressure head

$p = \rho g h$

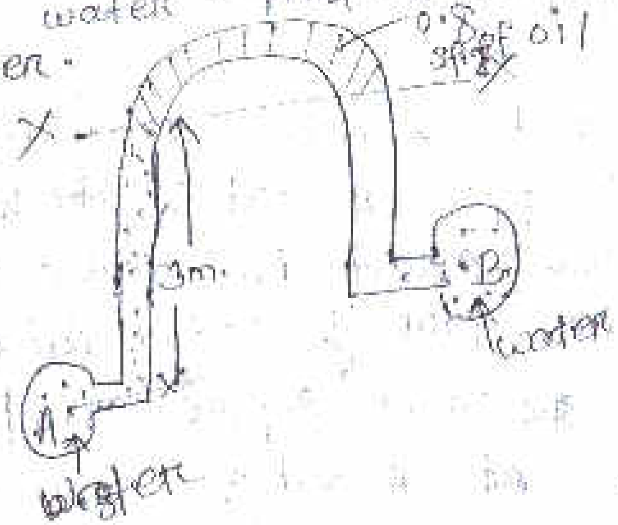
$h = \frac{p}{\rho g}$

$= 2m \times \rho g = \quad N/m^2$ Ans

$= 2m \times 1000 \times 9.81 = 19620 \ N/m^2$

$m \times \frac{kg}{m^2} \times \frac{m}{sec^2}$

20 water is flowing through different pipe to which an inverted differential manometer having an oil sp. gravity 0.8 is connected. The pressure head in the pipe A is 3 m of water. Find the pressure in pipe B. for manometer.




Soln $P_A$ = 3 m of water

$\Rightarrow \frac{P_A}{\rho g}$ = 3 m of water

$\Rightarrow \frac{P_A}{1000 \times 9.81} = 3$

$P_A = 1000 \times 9.81 \times 3 = 29430 \, N/m^2$

Taking XX datum pressure below left limb,

$P_A - 1000 \times 9.81 \times 0.3$

$= 29430 - 2943 = 26487 \, N/m^2$ —— (I)

pressure below x-x in right limb

$P_B - 1000 \times 9.81 \times 0.1 - 800 \times 9.81 \times 0.12$

$P_B - 1922.76$ —— (II)

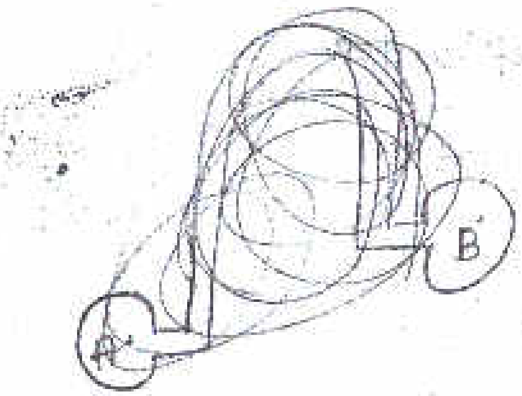
eqn (I) = eqn (II) at datum line

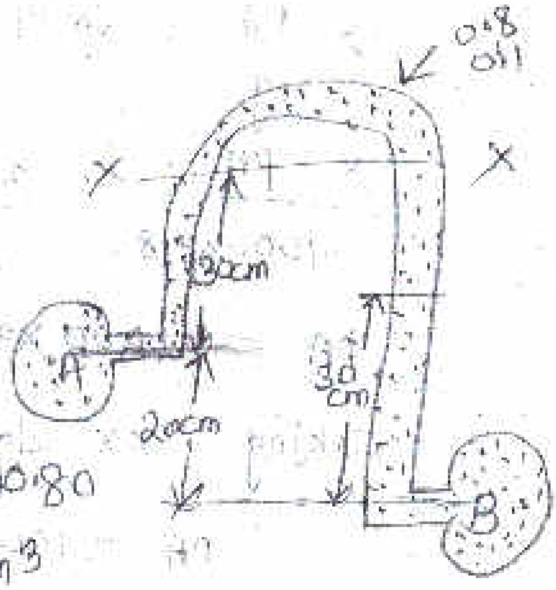
$PB - 1922.76 = 26487$

$\Rightarrow PB = 26487 + 1922.76$

$= 28409.76 \ N/m^2$

30 An inverted u-tube manometer connected to two pipes A and B which convey water. The fluid in manometer is oil of specific gravity 0.8. For the manometer reading shown in figure. Find pressure difference at A and B






Soln Specific gravity of oil = 0.80

density of oil = $800 \ kg/m^3$

Difference in oil in two limb $= (30+20) - 30$

$= 20 \ cm$

Taking x-x as datum line.

Pressure in the left limb below x-x

$PA - 1000 \times 9.81 \times 0.3 = PA - 2943$ — (1)

pressure in the right limb below x-x

$P_B - 1000 \times 9.81 \times 0.3 - 800 \times 9.81 \times 0.2$

$P_B - 2943 - 1569.6$ ——— (ii)

At datum pressure in left limb = pressure in right limb i.e eqⁿ (i) = (ii)

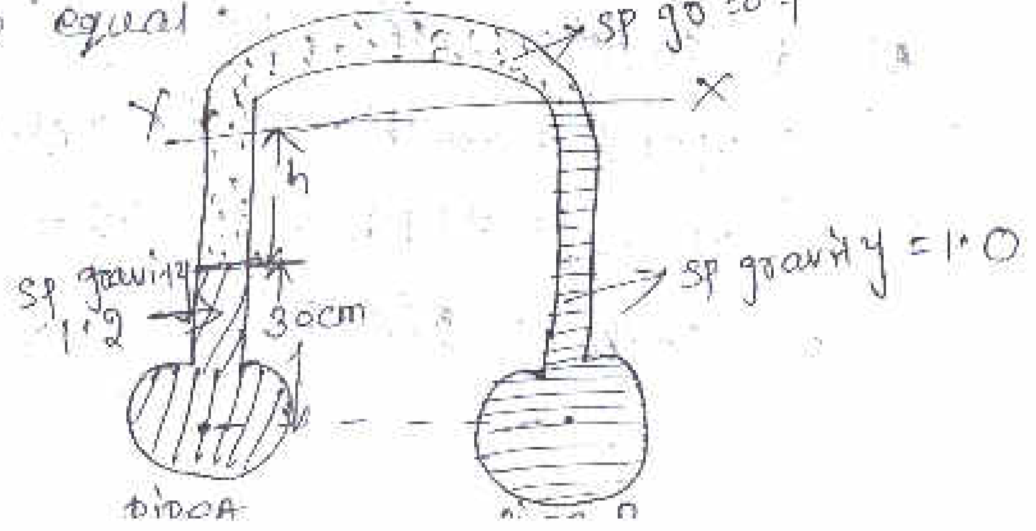
$P_A - 2943 = P_B - 2943 - 1569.6$

$\Rightarrow P_B - P_A = -\cancel{2943} + \cancel{2943} + 1569.6$

$= 1569.6 \ N/m^2$

22 Jun 2021

**4Q** Findout differential reading 'h' of an inverted U-tube manometer containing oil of specific gravity 0.7 as manometeric fluid when connected across pipes A and B as shown in fig. below conveying liquid of specific gravities 1.2 and 1.0 and immiscible with manometeric fluid. Pipes A and B are located at the same level and assume the pressure A and B are equal.



23 June 2021

Sol<sup></sup>

Let x-x taken as datum line.

Let PA = pressure at A

PB = pressure at B

Density of liquid in pipe A = SP gravity x $\rho_w$

$= 1.2 \times 1000 = 1200\ kg/m^3$

density of liquid in pipe B = $1 \times 1000$

$= 1000\ kg/m^3$

S.p gravity of oil = 0.7

density of oil = $0.7 \times 1000 = 700\ kg/m^3$

pressure below x-x as datum line pressure in left limb

$PA - 1200 \times 9.81 \times 0.3 + 700 \times 9.81 \times h$

$PA = 3531.6 + 6867h$ —— ①

pressure in Right limb

$PB - 1000 \times 9.81 \times (0.3 + h)$

$PB = 1000 \times 9.81 \times 0.3 + 1000 \times 9.81 \times h$

$PB = 2943 + 9810h$ —— ②

A/c to equation PA = PB

$\Rightarrow 3531.6 + 6867h = 2943 + 9810h$

$\Rightarrow 9810h + 6867h = 3531.6 - 2943$

$\Rightarrow 2943h = 588.6$

$\Rightarrow h = \frac{588 \cdot 6}{2943}$

$\Rightarrow h = 0.2\ m.$

$h = 20\ c \cdot m$

24 June 21 OMM SAI RAM

## Pressure Exerted on an Immersed surface

### Hydrostatic forces on surfaces :-

→ Here liquids and gases are at rest condition.

→ At rest condition means, there will be no relative motion between the adjacent or neighbouring fluid layers

→ So the velocity gradient is equal to zero. So that the shear stress will also zero.

Then the forces acting on fluid particles are :-

→ due to pressure acting normal to the surface

→ due to gravity (or self wt of particle)

## Total pressure

It is defined as the ~~pressure~~ force exerted by the static fluid on a surface either plane or curved when the fluid comes in contact with the surfaces.

→ It always acts normal to the surface.

**Centre pressure :-** It is defined as the point of application of the total pressure on the surface.

→ There will be four case of submerged surfaces on which the total pressure force and centre of pressure is to be determined.

The submerged surfaces may be

(i) vertical plane surface

(ii) Horizontal plane surface

(iii) Inclined plane surface

(iv) Curved surface

25 June 2021

**Vertical plane surface submerged in liquid:-**

Consider any arbitary shape immersed in liquid.

$A$ = Total area of the surface

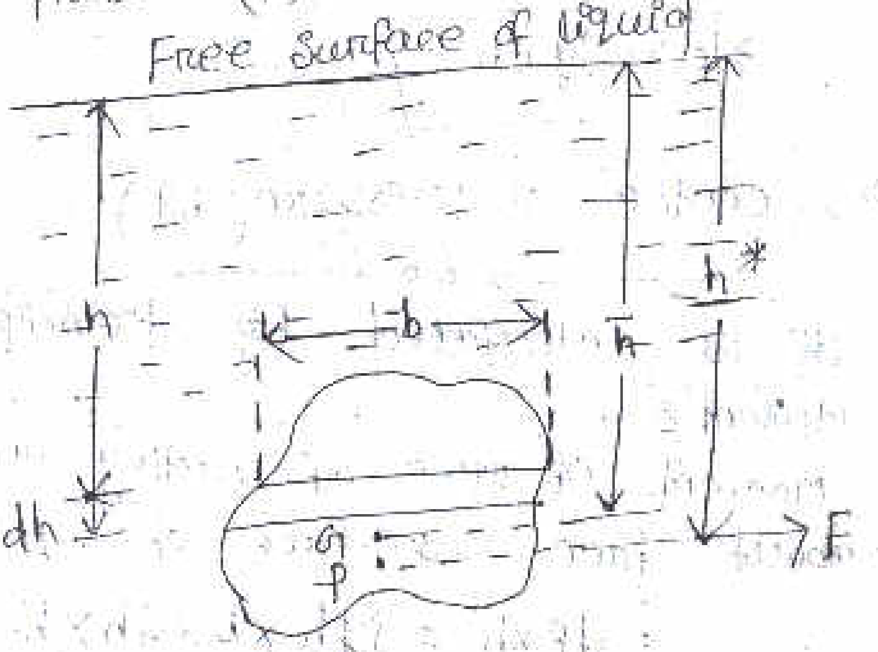
$\bar{h}$ : Distance of C.G of area from free surface of liquid.

G = Centre of gravity of plane surface

P = Centre of pressure

$h^*$ = Distance of centre of pressure from the free surface of liquid

~~Total~~ ~~pressure~~ (F)




(a) Total pressure (F)

consider a strip of thickness 'dh' and width 'b' at a depth of 'h' from free surface of liquid pressure intensity on the strip (p) = $\rho gh$

area of strip = $b \times dh$

Total force on the strip (dF) = p × Area

$= \rho gh \times b \times dh$

∴ Total pressure force on the whole surface

$\Rightarrow F = \rho g \int b \times h \times dh$

But $\int b \times h \times dh = \int h \times dA$

= Moment of surface area about the free surface of liquid.

= Area of surface × Distance of C.G from the free surface

$= A \times \bar{h}$

$\Rightarrow \boxed{F = \rho g A \bar{h}}$

(b) Centre of pressure ($h^*$)

It is calculated by Principle of Moments - Moment of force of acting on a strip about free surface of liquid

$= dF \times h = \rho g h \times b \times dh \times h$

Sum of moments of all such forces about free surface of liquids

$= \int \rho g h \times b \times dh \times h$

$= \rho g \int b \times h \times dh \times h$

$= \rho g \int b h^2 \cdot dh = \rho g \int h^2 \cdot dA$

$\int h^2 \cdot dA = \int b h^2 \cdot dA$

Sum of moment about free surface

$= \rho g I_0$ —— (1)

From principle of moments of force 'F' about free surface of liquid

$$= F \times h^* \quad \text{——} \quad (2)$$

Equating eq$^n$ (1) & (2)

$$F \times h^* = \rho g I_0 \quad (\text{But } F = \rho g A \bar{h})$$

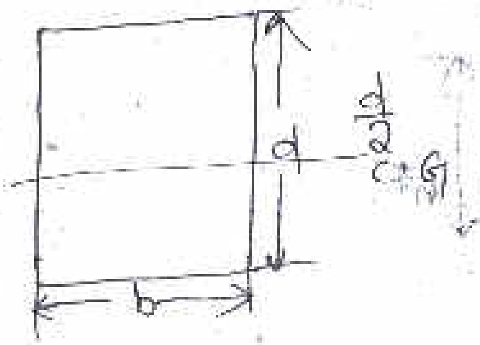
$$h^* \times \rho g A \bar{h} = \rho g I_0$$

$$\Rightarrow h^* = \frac{\rho g I_0}{\rho g A \bar{h}} = \frac{I_0}{A \bar{h}}$$

By theorem of parallel axis theorem

$$I_0 = I_G + A\bar{h}^2$$

So $\boxed{h^* = \frac{I_G + A\bar{h}^2}{A\bar{h}} = \frac{I_G}{A\bar{h}} + \bar{h}}$




centre of gravity $\bar{h} = \frac{d}{2}$

$$I_G = \frac{bd^3}{12}$$

$$A = b \times d$$

Centre of pressure $(h^*) = \frac{I_G}{A\bar{h}} + \bar{h} = \frac{\frac{bd^3}{12}}{b \times d \times \frac{d}{2}} + \frac{d}{2}$

$$\frac{bd^3}{12} \times \frac{2}{bd^2} + \frac{d}{2}$$

$$h^* = \frac{d}{6} + \frac{d}{2} = \frac{d + 3d}{6} = \frac{2d}{3}$$

1 Q A rectangular plane surface is 2m wide and 3 m deep. It lies in vertical plane in water Determine the total pressure and position of centre of pressure on the plane surface when its upper edge is horizontally and

(a) Coincides with free water surface.

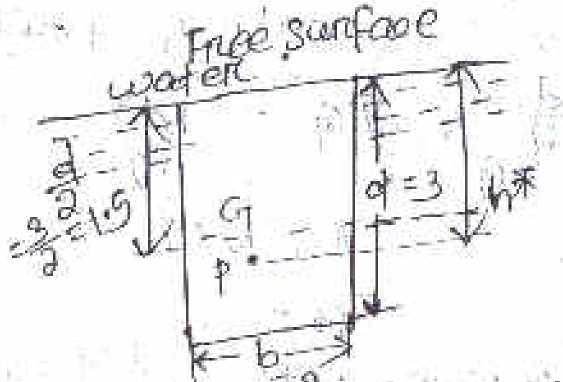
(b) 2.5m below the free water surface.

Soln Given that :-

wide(b) = 2m

Deep (d) = 3m.

(a) upper edge coincides with the water surface




$\bar{h} = 1.5m.$

$Area = b \times d = 2 \times 3 = 6m^2$

$F = \rho g A\bar{h}$

$= 1000 \times 9.81 \times 6 \times 1.5m$

$= 88290\ N$

Depth of centre of pressure

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

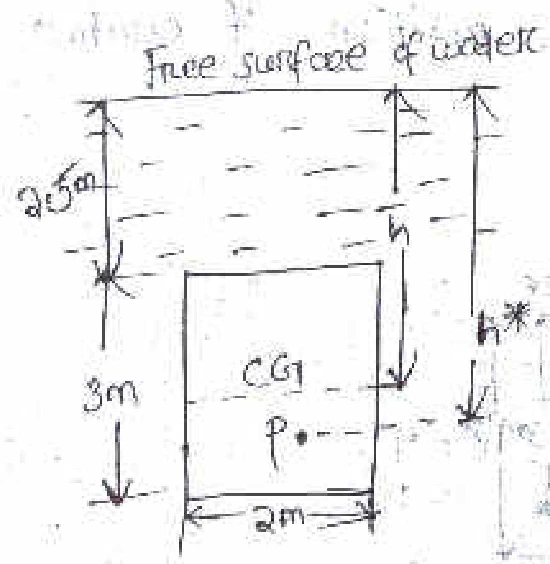
$$I_G = \frac{bd^3}{12} = \frac{2\times 3^3}{12} = 4.5m$$

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

$$= \frac{4.5}{6\times 1.5} + 1.5 = \frac{1}{2} + \frac{3}{2}$$

$$= \frac{1+3}{2} = \frac{4}{2} = 2m$$

(b)




$$F = \rho g A\bar{h}$$

$$A = 2\times 3 = 6m^2$$

$$\bar{h} = \frac{3}{2} + 2.5 = 4m$$

$$F = \rho g A\bar{h}$$

$$= 1000\times 9.81\times 6\times 4$$

$$= 235440\ N$$

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

$$I_G = \frac{bd^3}{12} = \frac{2\times 3^3}{12} = 4.5m$$

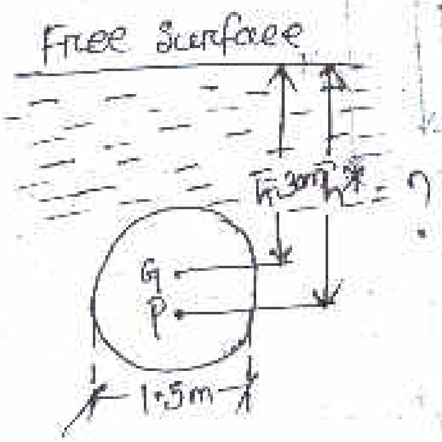
$= \dfrac{4.5}{6 \times 4} + 4$

$= 4.1875m.$

5 July 2021

**10** Determine the total pressure on a circular plate of diameter 1.5 m which is placed vertically in water in such a way that the centre of plate is 3m below the surface of water. Find the position of centre of pressure.

Soln




Given data :- Dia of plate $d = 1.5m$.

Area of plate $(A) = \frac{\pi}{4} \cdot d^2$

$= \frac{\pi}{4}(1.5)^2 = 1.767\ m^2$

Total pressure $(P) = \rho g A \bar{h}$

$= 1000 \times 9.81 \times 1.767 \times 3.0$

$= 52002.81\ N$

position of centre of pressure

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h} \qquad I_G = \frac{\pi d^4}{64}$$

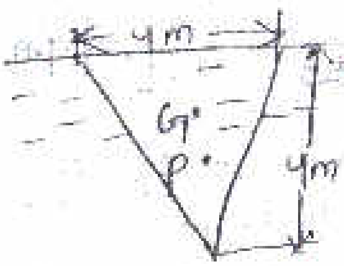
$$= \frac{\pi}{64} \times (1.5)^4$$

$$= 0.248 \text{ m}$$

$$h^* = \frac{0.2485}{1.767 \times 3.0} + 3.0$$

$$= 0.468 + 3.0 = 3.0468 \text{ m.}$$ Ans

Q2




Determine the total pressure and centre of pressure of an isosceles triangular plate of base 4m and altitude 4m. when it is immersed vertically in an oil of sp gravity 0.9. The base of plate coincides with the free surface.

Soln

Base of plate (b) = 4m.

Altitude / Height (h) = 4m.

Area (A) $= \frac{1}{2} b \times h = \frac{1}{2} \times 4 \times 4 = 8 \text{ m}^2$

sp gravity of oil = 0.9

Density of oil ($\rho$) = 900 kg/m$^3$

Distance of C.G from the free surface of oil = $\frac{1}{3}h = \frac{1}{3} \times 4 = 1.33$ m.

Total pressure (F) = $\rho g A \bar{h}$

$= 900 \times 9.81 \times 8 \times 1.33$

$= 9597.6$ N

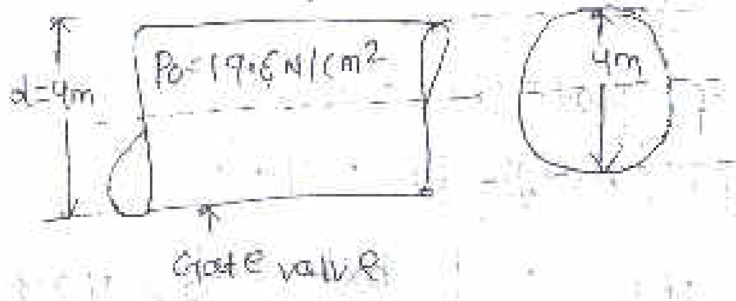
Centre of pressure ($h^*$) = $\frac{I_G}{A\bar{h}} + \bar{h}$

$I_G = \frac{bh^3}{36} = \frac{4 \times 4^3}{36} = 7.11$ m$^4$

$\Rightarrow h^* = \frac{7.11}{8 \times 1.33} + 1.33 = 1.99$ m.

5 Aug 2021

1-Q A pipe line which is 4m diameter contains a gate valve. The pressure at the centre of pipe is 19.6 N/cm$^2$. If the pipe is filled with oil of specific gravity 0.87 find the force exerted by the oil upon the gate and the position of centre of pressure.



Sol^n Given data

dia = 4m.

pressure at the centre of pipe ($p_0$) = $19.6\ N/cm^2$

$= 19.6 \times 10^4\ N/m^2$

sp. gravity of oil = 0.87

density of oil = $0.87 \times 1000$

$= 870\ kg/m^3$

pressure head at the centre of pipe

$$\bar{h} = \frac{p_0}{\rho g} = \frac{19.6 \times 10^4}{870 \times 9.81} = 22.988m.$$

The height of equivalent free oil surface from Centre of pipe = 22.988m.

now force exerted by the oil on the gate is

$$(F) = \rho g A \bar{h}$$

$$= 870 \times 9.81 \times \frac{\pi}{4} \times 4^2 \times 22.988$$

$$= 2465500\ N \quad = 2.465 KN.$$

position of centre of pressure

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

Here $I_G = \frac{\pi d^4}{64}$, $A = \frac{\pi}{4} d^2$, $\bar{h} = 22.988m.$

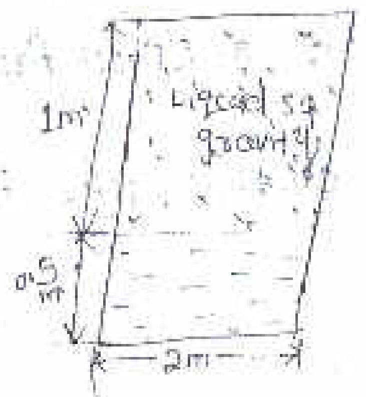
$$h^* = \frac{\frac{\pi}{64} d^4}{\frac{\pi}{4} d^2 \times \bar{h}} + \bar{h}$$

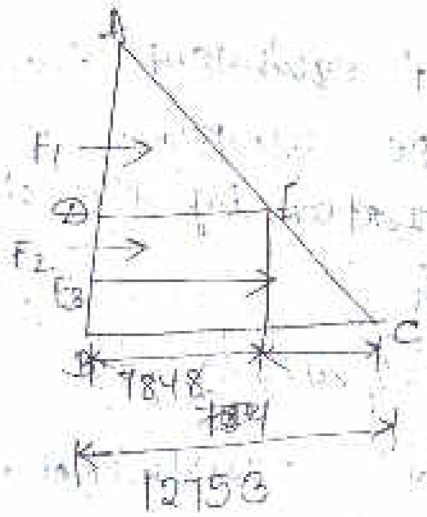
$$h^* = \frac{d^2}{16\bar{h}} + \bar{h} = \frac{(4)^2}{16 \times 22.988} + 22.988$$

$\Rightarrow h^* = 0.043 + 22.988$

$h^* = 23.031m$ <u>Ans</u>

2 Q. A tank contains water upto a height of 0.5m above the base. An immiscible liquid of sp. gravity 0.8 is filled on the top of water upto 1m height calculate.
(i) Total pressure on one side of tank.
(ii) The position of centre of pressure for one side of tank, which is 2m wide.




<u>Given data :-</u>

depth of water = 0.5m

depth of liquid = 1m

specific gravity of liquid = 0.8

Density of liquid ($\rho_1$) = $0.8 \times 1000 = 800 kg/m^3$

Density of wate ($\rho_2$) = $1000 kg/m^3$

width of tank (b) = 2m.

1. Total pressure on one side of tank :-

Intensity of pressure at top i.e

$\rho g P_A = 0$

Intensity of pressure at 'D'

$P_D = \rho_1 g h_1$

$= 800 \times 9.81 \times 1.0 = 7848 \, N/m^2$

Intensity of pressure at based in i.e

$\rho_1 g h_1 + \rho_2 g h_2$

$= 800 \times 9.81 \times 1 + 1000 \times 9.81 \times 0.5$

$= 12755 \, N/m^2$

Now force $F_1$ = Area of ADE $\times$ width of tank

$= \frac{1}{2} \times AD \times DE \times 2.0 m$

$= \frac{1}{2} \times 1 \times 7848 \times 2 = 7848 \, N$

$F_2$ = Area of $\Delta$ EFC $\times$ width of tank

$= \frac{1}{2} EF \times FC \times 2.0$

$= \frac{1}{2} \times 0.5 \times 4905 \times 2.0$

$= 2452.5 \, N$

Total force (F) $= F_1 + F_2 + F_3$

$= 7848 + 7848 + 2452.5$

$= 18148.5 N$

(ii) Taking moment about 'A' of all three forces

$$F \times h^* = F_1 \times \frac{2}{3} AD + F_2 \left(AD + \frac{1}{2} BD\right) + F_3 \left(AB + \frac{2}{3} BD\right)$$

$$\Rightarrow 18148 \times h^* = 7848 \times \frac{2}{3} \times 1 + 7848 \left(1.0 + \frac{0.5}{2}\right) + 2452.5 \left(1.0 + \frac{2}{3} \times 0.5\right)$$

$$\Rightarrow 18148.5 \times h^* = 5232 + 9810 + 3270$$

$$\Rightarrow h^* = \frac{18312}{18148.5}$$

$$= 1.009 \text{ From top.}$$

# Kinematics of flow & Ideal flow

7 Aug 2021

Kinematics is defined as that branch of science which deals with motion of particles without considering the forces causing motion

Types of fluid flow:-

(i) steady and unsteady flow
(ii) uniform & non-uniform flow
(iii) Laminar & Turbulent flow
(iv) compressible & incompressible flow
(v) Rotational & Irrotational flow
(vi) one, two & three-dimensional flow.

Steady flow:-
It is defined as that type of flow in which the fluid characteristics like velocity, pressure & density etc. donot change with time.

$$\frac{\partial V}{\partial t} = 0 \qquad \frac{\partial p}{\partial t} = 0, \quad \frac{\partial \rho}{\partial t} = 0$$

Unsteady flow:- It is defined as that type flow in which the fluid characteristics changes with respect to time.

$$\frac{\partial V}{\partial t} \neq 0, \quad \frac{\partial p}{\partial t} \neq 0, \quad \frac{\partial \rho}{\partial t} \neq 0$$

Uniform flow:- It is defined as that type of flow in which the velocity at any given time doesnot change with respect to distance / space.

$$\left(\frac{\partial v}{\partial s}\right)_{t = \text{Const}} = 0$$

$\partial v$ = change in velocity

$\partial s$ = Length of flow in direction of S.

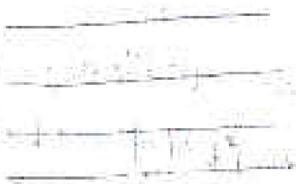
Non-uniform flow :- In this flow, the velocity with a given time change with respect to space.

$$\left(\frac{\partial v}{\partial s}\right)_{t = \text{const}} \neq 0$$

Laminar & Turbulent flow :-

Laminar flow

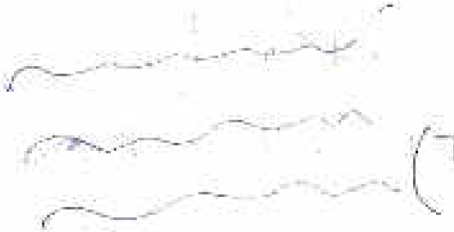
Laminar flow is defined as that type of flow in which fluid characteristics move along well defined path or streamline and all the streamlines are straight and parallel.



(Laminar fluid flow)

Turbulent flow

Turbulent flow is defined as that type of flow in which the fluid particles move in a zig-zag way. Due to movement of fluid particles in a zig-zag way, eddies formation takes place because of high energy loss.



(Turbulent fluid flow)

non - dimensional no.

$$\boxed{\text{Reylond's no} = \frac{\rho V D}{\nu}}$$

Re < 2000 (Laminar flow)

Re > 4000 (Turbulent flow)

2000 > Re > 4000 (Transitional flow)

where

$\rho$ = density of fluid

V = velocity of fluid

D = Diameter of pipe

$\nu$ = Kinematic viscosity of liquid.

Compressible & Incompressible flow

Compressible flow - Is that type of flow in which the density of fluid changes from one point to point or in other words, density ($\rho$) is not constant for fluid.

$$\boxed{\rho \neq \text{constant}}$$

Incompressible flow Is that type of flow in which the density of fluid changes from one point to point or in other words density($\rho$) is constant for fluid.

$$\boxed{\rho = \text{constant}}$$

## Rotational and Irrotation flow :-

Rotational flow is that type of flow in which fluid particles flowing along a streamlines and also rotate about their own axis.

**Irrotational flow :-** Fluid particles while flow along the streamlines, do not rotate about their own axis.

## Rate of Discharge (or) flow (Q).

It is defined as the quantity of fluid flowing per second through a section of pipe or a channel.

> For liquids, unit of Q = $m^3/sec$ or lit/sec

> For gas, unit of Q = kgf/s or N/s

Consider a liquid flow in a pipe,

$$\boxed{Q = AV}$$

A = area of cross section of pipe.

V = average velocity of fluid across the section.

## Continuity Equation :-

Based on conservation of mass thus, for a fluid flowing through the pipe at all the cross-section, the quantity of fluid flowing per second is constant.

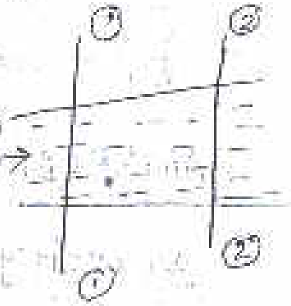
Consider two section of pipe.

$V_1$ = avg. velocity at section ①-①

$A_1$ = Area of pipe at section ①-①

$V_2$ = avg. velocity at section ②-②

$A_2$ = Area of pipe at section ②-②




Rate of flow at section ①-①

$\rho_1 A_1 V_1$

Rate of flow at section ②-②

$\rho_2 A_2 V_2$

A/c to conservation of mas

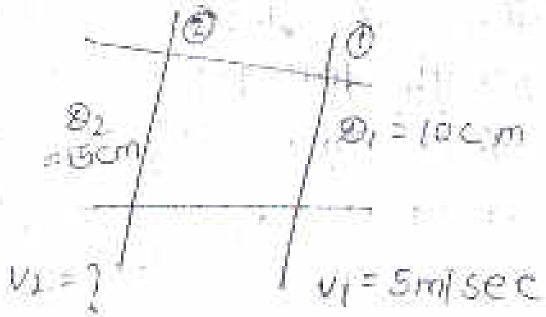
$$\boxed{\rho_1 A_1 V_1 = \rho_2 A_2 V_2} \quad \text{--- ①}$$

for compressible fluid for incompressible fluid continuity equal.

$$\boxed{A_1 V_1 = A_2 V_2} \quad (\rho_1 = \rho_2)$$

10 Aug 2021

1.Q




Find discharge through the pipe if the velocity of water is flowing through the pipe at section ① is 5m/sec. Determine the velocity at section ② also ?

Son Given data :-

$D_1 = 10c.m = 0.1m.$

$Area(A_1) = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (0.1)^2 =$

velocity at secn ①-① $(V_1) = 5m/sec$

Diameter at section ②-② $(D_2) = 15c.m = 0.15m.$

$A_2 = \frac{\pi}{4} \times (0.15)^2.$

A/c continuity eqn

$Q = A_1V_1 = A_2V_2$

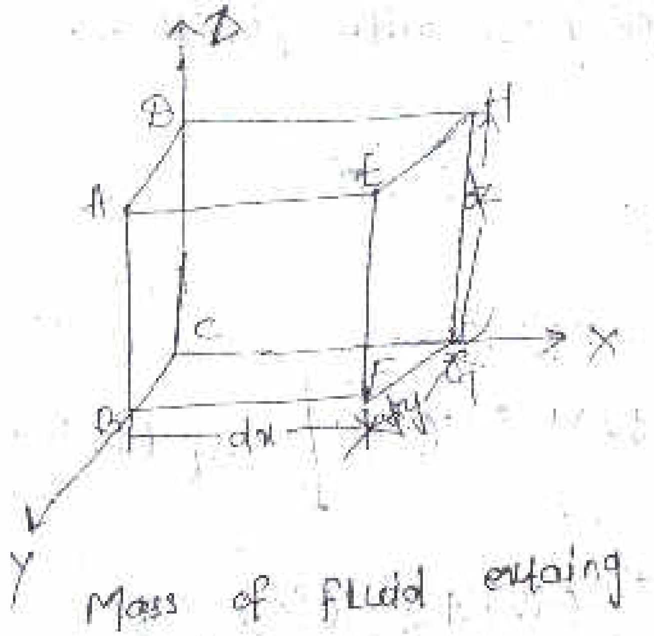
$Q = A_1V_1$

$Q = \frac{\pi}{4}(0.1)^2 \times 5m/sec$

$= 0.03927\ m^3/sec$

$A_1V_1 = A_2V_2$

$\Rightarrow V_2 = \frac{A_1V_1}{A_2} = 2.22m/sec$




Mass of fluid entering the face of ABCD per second,

$= \rho \times$ velocity $\times$ Area of ABCD

$= \rho \times u \times dy \times dz$

Mass of fluid leaving the face EFGH per second $= \rho u dy dz + \frac{\partial}{\partial x}(\rho u dy dz) dx$

Gain of mass in x-direction

= Mass through ABCD − Mass through EFGH

$= \rho u dy \cdot dz - \frac{\partial}{\partial x}(\rho u dy dz) \cdot dx$

$= \frac{-\partial}{\partial x}(\rho u \cdot dy dz) dx$

$= \frac{-\partial}{\partial x}(\rho u) \cdot dx \cdot dy \cdot dz$

Similarly, gain of mass in y-direction

$= -\frac{\partial}{\partial y}(\rho v) \cdot dx \cdot dy \cdot dz$

in z-direction $= \frac{-\partial}{\partial z}(\rho w) \cdot dx \cdot dy \cdot dz$

Net gain of mass $= -\left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right] dx\, dy\, dz$ —— (1)

Rate of increase of mass with fluid element w.r.t time.

$$\frac{\partial}{\partial t}(\rho \, dx\, dy\, dz) \text{ ——— } (2)$$

equating eqⁿ (i) & eqⁿ (ii)

$$-\left[\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)\right]=\frac{\partial}{\partial t}(\rho \cdot dx \cdot dy \cdot dz)$$

$$\Rightarrow \frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)=0$$

$$\Rightarrow \boxed{\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)=0}$$

This eqⁿ is applicable for

(i) steady & unsteady flow

(ii) uniform & non-uniform flow

(iv) compressible & incompressible flow.

for steady flow, $\frac{\partial \rho}{\partial t}=0$, so the eqⁿ is

$$\boxed{\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)=0}$$

If the fluid incompressible then $\rho$ = constant

$$\boxed{\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0}$$ continuity eqⁿ in three-dimension

Continuity eqⁿ is 2-Dimensions :-

$$\boxed{\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0}$$

Velocity & Acceleration

$V \rightarrow$ Resultant velocity

$u, v, w \Rightarrow$ velocity component in $x, y$ & $z$ direction

$u = f_1(x, y, z, t)$

$v = f_2(x, y, z, t)$

$w = f_3(x, y, z, t)$

Resultant velocity $(V) = u\hat{i} + v\hat{j} + w\hat{k}$

$\Rightarrow \boxed{V = \sqrt{u^2 + v^2 + w^2}}$

Let $a_x, a_y, a_z$ the total acceleration in $x, y$ & $z$ directions respectively

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial u}{\partial y} \cdot \frac{\partial y}{\partial t} + \frac{\partial u}{\partial z} \cdot \frac{\partial z}{\partial t} + \frac{\partial u}{\partial t}$$

But we know $\frac{\partial x}{\partial t} = u$, $\frac{\partial y}{\partial t} = v$, $\frac{\partial z}{\partial t} = w$

$$a_x = \frac{du}{dt} = u \cdot \frac{\partial u}{\partial x} + v \cdot \frac{\partial u}{\partial y} + w \cdot \frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

similarly

$$a_y = \frac{dv}{dt} = u \frac{\partial v}{\partial x} + v \cdot \frac{\partial v}{\partial y} + w \cdot \frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$a_z = \frac{dw}{dt} = u \frac{\partial w}{\partial x} + v \cdot \frac{\partial w}{\partial y} + w \cdot \frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

For steady flow, $\frac{\partial V}{\partial t} = 0$

then $\frac{\partial u}{\partial t} = 0$, $\frac{\partial v}{\partial t} = 0$, $\frac{\partial w}{\partial t} = 0$

Hence acceleration in $x$, $y$ & $z$ directions.

$$a_x = \frac{du}{dt} = u\cdot\frac{\partial u}{\partial x} + v\cdot\frac{\partial u}{\partial y} + w\cdot\frac{\partial u}{\partial z}$$

$$a_y = \frac{dv}{dt} = u\cdot\frac{\partial v}{\partial x} + v\cdot\frac{\partial v}{\partial y} + w\cdot\frac{\partial v}{\partial z}$$

$$a_z = \frac{dw}{dt} = u\cdot\frac{\partial w}{\partial x} + v\cdot\frac{\partial w}{\partial y} + w\cdot\frac{\partial w}{\partial z}$$

Total Acceleration $(A) = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

$$A = \sqrt{a_x^2 + a_y^2 + a_z^2}$$